SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 9, No. 3, October 2012, 361-375

UDK: 621.391:004.7]:519.213 **DOI: 10.2298/SJEE1203361M**

Calculating the Required Number of Bits in the Function of Confidence Level and Error Probability Estimation

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Abstract: This paper proposes the calculation of the required number of bits transmitted in the system, in order to achieve the desired level of confidence. Proposed solution is based on the selected value for the bit error probability, and statistical confidence levels. In addition, required error probability for telecommunications protocols and data transfer protocols is discussed, overview of the BER (Bit Error Ratio) testing equipment performance is provided, and compromise of testing time in relation to the statistical level of confidence depending on BER is examined.

Keywords: Digital telecommunication system, Estimating error probability, Statistical confidence level, Poisson theorem.

1 Introduction

In digital communications systems, the ultimate function of the physical layer is to as quickly and as accurately as possible transfer data bits in the media. Data can be transmitted over copper cable, optical fiber or free space. Two basic measures of physical performance levels are related to the speed at which data can be transmitted (*Data Rate*) and data integrity when they arrive at the destination. The primary measure of data integrity is called the *Bit Error Ratio*, or BER.

For digital communications systems, BER can be defined as the estimated probability of error. This means that any bit that is transmitted through the system may be falsely accepted. So, transferred "one" will be accepted as "zero", or transferred to a "zero" will be received as a "one". In practical testing, BER is measured by transferring a finite number of bits through the system and counting incorrectly received bits. The ratio of incorrectly received bits and the total number of transferred bits is called *Bit Error Ratio* (BER). Quality of BER assessment is increased if the total number of transmitted bits increases. In the limiting case, when the number of transmitted bits tends infinity, the BER becomes perfectly estimated the actual probability of error.

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In some articles, BER is called as the *Bit Error Rate* instead of the *Bit Error Ratio*. Most of the wrong bits in real systems are the result of random noise, and occur at random in contrast to the uniformly distributed noise. BER assessment is carried out in such a way, that the ratio of number of incorrect and total transferred bits is formed, so it is more correct to use the word *ratio*, than word *rate*.

Depending on the exact number of bits (data pattern) conveyed through the system, there may be a different number of wrong bits. Patterns that contain long strings of *Consecutive Identical Digits* (CID) may contain significant low frequency spectral components that may be outside of the bandwidth of the system, causing deterministic jitter and other distortions in the signal, [1]. These effects depend on the data patterns and may increase or decrease the probability that wrong bits are transferred. This means that it is possible to get different results, when the BER is tested using various sequences of bits (data patterns).

Detailed analysis of the effects that depend on data patterns is not the topic of this work, but it is enough to keep in mind the importance of accession of the specific data patterns to the specifications of BER and test results.

Statistical level of confidence, SLC is defined as the probability, based on a set of measurements, that the actual probability of events is greater than some defined value. (For purposes of this definition, the real probability means the probability which is determined as the limit when the number of events n tends to infinity.) When applying this definition to the BER, the definition of SLC can be explained as (based on detection of errors when n bits are transferred) the actual probability of the event, BER, which is greater than a defined value γ.

Based on the selected value for the probability of bit errors, BER, and level of SLC, the required number of transferred bits n is determined, which must be transmitted through the system to achieve the desired level of confidence. We then calculated the time of testing, and reduced testing time needed to complete the test of system in order to get the desired level of confidence, depending on the BER. The value of BER, which is especially interesting for us, is about 10^{-9}, as in [2] in Fig. 6.

2 Definition of the Predicted Error Probability BER

In most digital communication protocols, BER is defined by two values. Telecommunication protocols, such as SONET (*Social Networking*), usually require a *BER* of one bit error in 10^{10} bits (BER = $1/10^{10} = 10^{-10}$) using a long pseudo-random series of bits. In contrast, protocols for data transmission over fiber optic and Ethernet channels typically require BER of less than 10^{-12} bits (BER < 10^{-12}) using short sequences of bits. In some cases, the specification of the system requires BER of 10^{-16} bits or less (BER $\leq 10^{-16}$).

It is important to note that the BER is essentially statistical average and it is only necessary to transfer a great enough number of bits. It is possible, for example, that it exists more than one error within the group of, for example, 10^{10} bits, and that the BER specification of 10^{-10} is still satisfied. The BER of 10^{-10} will be satisfied when the total number of transferred bits is much higher than 10^{10}. It can also happen that there is less than 1 error in 10^{10} bits in the subsequent series of transmitted bits. Alternatively, it is possible to have zero error bits within the group of 10^{10} bits, and that BER is still higher than the 10^{-10} (BER > 10^{-10}), if there are more errors in subsequent groups of transferred bits.

Considering these examples, it is clear that a test which determines the BER which is better than 10^{-10} must be performed so that much more than 10^{10} bits are sent in order to obtain accurate and reproducible measurement. On the basis of the previous consideration, we can now determine the number of bits which must be transmitted through the system in order to show that the BER is less than or equal to specified value (Section 4).

3 Equipment and Testing Procedures

Standard method of BER testing in one system uses a pattern generator and detector of errors, Fig. 1. Pattern generator transfers test series to the system under test. Error detector can independently generate the same test sequence, or it can receive it from the pattern generator. Pattern generator also provides the synchronization clock signal for the error detector. Error detector performs bit-for-bit comparison between the data obtained from the system under test and the data generated by the pattern generator. Any difference between these two sets of data is counted as a bit error, BER.

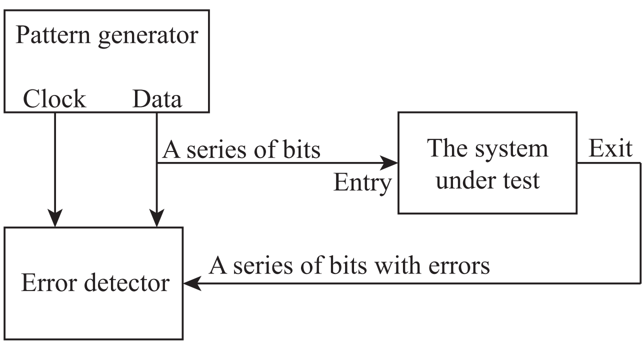

Fig. 1 – *Test equipment and system in which* BER *is tested.*

As noted in the previous section, the standards for digital communications generally specify the data pattern used for BER testing. The test pattern is usually chosen to emulate the type of data that are expected during normal operation, or, in some cases, the sample that is particularly inconvenient for the system can be selected, in order to obtain the worst case of testing. An aim of using the pattern is to approximate the random data. The pattern is called pseudo random bit sequence (*Pseudorandom Binary Sequence*, PRBS), and it based on standardized algorithms of generation. *PRBS* sequences are classified according to the length of the series and usually are named according to these lengths as "2^7–1" (pattern length = 127 bits) or "2^{23}–1" (pattern length = 8,388,607 bits). Other sequences simulate encoded/encrypted data or inconvenient sequence of data, and the known sequences are K28.5, (they use fibre channel and Ethernet) etc, [1]. Standard available pattern generators include standard built-in patterns, and the ability to create custom data patterns.

In order to compare accurate bits obtained from the pattern generator to the bits received from the system being tested, the error detector must be synchronized on both patterns of bits and synchronization has to compensate time delays through the system under test. Clock signal from pattern generator provides synchronization of bits obtained from pattern generator. Error detector adds variable time delay to the signal generated in the pattern generator to allow synchronization of bits received from the system under test. Variable time delay is adjusted to minimize bit errors.

4 Determination of the Required Numbers of Bits

In well-designed systems, BER performance is limited by random noise and/or by random jitter. The result is that errors occur in a random (unpredictable) time. The errors can be bursty or they can appear separately. Accordingly, the number of errors that occur during operation of the system is a random variable that can not be accurately predicted. The real answer to the question how many bits must be transmitted through the system for a perfect *BER* test is so: unlimited number (essentially infinite).

From a practical BER testing is required that the time of test is finite. We must accept worse value of the estimation than the perfect estimation. As previously stated, if the quality of BER assessment increases, then, also, the total number of transmitted bits increases. The problem is how to quantify the improvement of the quality assessment, so that it can be determined how many bits must be transmitted to obtain the desired quality of assessment. This can be done using the concept of SLC. In statistical meaning, the level of confidence in the value of error probability ($P(e)$), or BER, can be defined as a probability (which is based on the detected number of errors, (e), in n transmitted bits) that the true $P(e)$ or BER is less than the mentioned ratio γ. (For the purpose of this definition, the true $P(e)$ means $P(e)$ which is measured, if the number of transferred bits is infinite). Mathematically, this can be expressed as:

$$\mathrm{SLC} = P\left[P(e) < \gamma \mid e, n\right], \tag{1}$$

where the SLC is the level of confidence in $P(e)$, $P[.]$ indicates "probability that", and $P(e)$ is the actual BER. As the level of confidence is, by definition, probability, its range of possible values is from 0 to 100%. When the BER can be determined for a certain level of confidence, then we can say that the SLC is percentage of confidence when the true BER is less than γ. Another interpretation is that, if always the same number of bits, n, is transmitted through the system and calculated number of detected errors, e, is repeated every time during testing, it can be expected as a result of the BER estimation e/n, that real BER will be less than γ for SLC percent of repeated testing, [3].

If we now consider equation (1), then the question at which we really want to know the answer is how to form equation (1), in order to be able to calculate how many bits must be transferred, to obtain the predicted BER, for a given level of confidence. In order to do this, we use the statistical method that includes the binomial distribution function and *Poisson* theorem.

4.1 Binomial distribution function

Calculation of the level of confidence is based on the binomial distribution function, which is described in most books obout statistics [4, 5]. Binomial distribution function can be expressed as:

$$P_n(k) = \binom{n}{k} p^k q^{n-k} \,, \tag{2}$$

where $\binom{n}{k}$ is defined as $\dfrac{n!}{k!(n-k)!}$.

The probability that k events (number of wrong bits) occurs in n attempts (n bits are transferred) is given in (2). In (2) p is the probability that event occurs in one attempt (a bit error), and q is the probability that this does not happen in one attempt (no error bits). Binomial distribution model has two possible outcomes, such as success/failure or error/no error. So, $p + q = 1$.

The function of the cumulative binomial distribution, for the probability $P(e)$ or BER, when less than N events occur in n attempts (or vice versa, that more than N events occur in n attempts), is given as:

$$\begin{aligned} P(e \le N) &= \sum_{k=0}^{N} P_n(k) = \sum_{k=0}^{N} \frac{n!}{k!(n-k)!} p^k q^{n-k} \,, \\ P(e > N) &= 1 - P(e \le N) = \sum_{k=N+1}^{n} \frac{n!}{k!(n-k)!} p^k q^{n-k} \,. \end{aligned} \tag{3}$$

Equations (1), (2) and (3), are presented graphically in Fig. 2. The following parameters were used to obtain a graphic in Fig. 2:

– $P_n(k)$ is defined in (3);

– $n = 10^8$, total number of events (total number of transmitted bits);

– k is the number of events that took place in n attempts (number of wrong bits);

– $p = 10^{-7}$, the probability that the event occurred in the considered attempt (bit error probability, BER);

– $q = 1 - 10^{-7}$, the probability that the event did not happen in the considered attempt (the probability that no bit error happened);

– $p + q = 1$;

– mean value, $\mu = nq$;

– variance, $\sigma^2 = npq$.

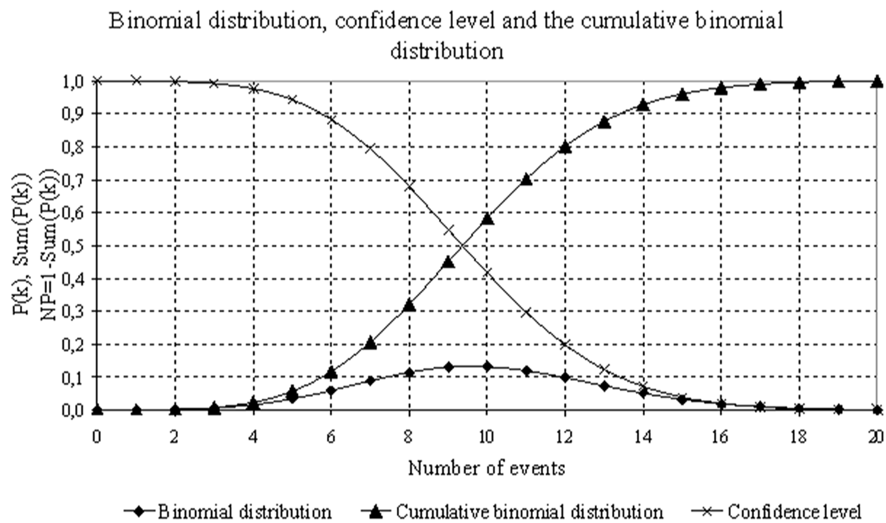

Fig. 2 – *Binomial distribution, confidence level and the cumulative binomial distribution in the function of number of errors.*

4.2. Application of binomial distribution function to calculate the level of confidence

In measuring the level of confidence, we begin by selecting the appropriate level of confidence and a hypothesis for the value p (bit error probability in the transmission of one bit). The selected p value is represented as p_h. In principle, these values were chosen according to constraints imposed by telecommunication systems (if the limit is $P(e) \le 10^{-10}$, we choose $p_h = 10^{-10}$, and level of confidence, for example, 90% or 99%).

Using the cumulative binomial distribution function, the level of confidence is defined as:

$$\mathrm{SLC} = P(e > N/p_h) = 1 - \sum_{k=0}^{N} \frac{n!}{k!(n-k)!} p_h^k (1-p_h)^{n-k}, \tag{4}$$

where SLC is the level of confidence, Fig. 2.

Now we use (4) to determine the probability $P(e > N/p_h)$, based on p_h, that more than N wrong bits will happen, when a total of n bits are transferred. If, during the actual testing, the number of wrong bits is less than N bits (and if $P(e > N/p_h)$ is large), one of the following two conclusions can be made [2]:

(a) we are fortunate in the selection of p_h,

(b) the actual value of p is less than p_h.

If the test is repeated several times and the number of measured wrong bits is constantly less than N, then is more and more secure the conclusion (b).

Value $P(e > N/p_h)$ defines the level of confidence in the conclusion (b). If $p_h = p$, there is a high probability, to detect more than N wrong bits. When less than N errors are measured, it is concluded that p is probably less than p_h, and definition of level of confidence gives the probability that the conclusion is correct. In other words, we can be sure that the value of $P(e)$, (where $P(e)$ is the actual bit error probability) is less than p_h for the level of confidence SLC%.

4.3. Determination of the required number of transferred bits, *n*

As noted above, when using the method of the level of confidence, in general we use hypothetical value of $p = p_h$ with the desired level of confidence, *SLC*. Then we solve (4), to determine how many bits, n, must be transferred through the system (with N or fewer errors). Solving (4), n can be very difficulty determined if certain approximations are not used.

Assuming $n{\cdot}p > 1$ (at least as many bits are transmitted as is the reciprocal value of the probability of wrongly transmitted bits) and k is the same order of magnitude as np, then *Poisson's* theorem, (5) from [1], provides conservative estimate for the binomial distribution function:

$$P_n(k) = \left(\frac{n!}{k!(n-k)!}\right) p^k q^{n-k} \xrightarrow[n \to \infty]{} \frac{(np)^k}{k!} \mathrm{e}^{-np}. \tag{5}$$

In (6) it is presented that (5) can be used to obtain approximation for the cumulative binomial distribution:

$$\sum_{k=0}^{N} P_n(k) \approx \sum_{k=0}^{N} e^{-n \cdot p} \frac{(np)^k}{k!}. \tag{6}$$

Combining (4) and (6), n can be calculated as:

$$\sum_{k=0}^{N} P_n(k) = 1 - \text{SLC}, \quad \text{(rearrangement of (4))},$$

$$\sum_{k=0}^{N} \frac{(np)^k}{k!} \mathrm{e}^{-np} = 1 - \text{SLC}, \quad \text{(utilisation of (6))},$$

$$-np = \ln\left[\frac{1-\text{SLC}}{\sum_{k=0}^{N} \frac{(np)^k}{k!}}\right], \tag{7}$$

$$n = -\frac{\ln(1-\text{SLC})}{p} + \frac{\ln\left(\sum_{k=0}^{N} \frac{(np)^k}{k!}\right)}{p}.$$

Now we can determine the total number of bits, n, which must be transmitted over the system to achieve the desired level of confidence, for the predicted probability of error $P(e)$ or BER, and the procedure is as follows [2]:

1. *The desired value for p_h, a hypothetical value for of p, is selected*. Value p_h is the probability of bit error that is being checked. If we want to show that $P(e) \le 10^{-12}$, then in (7) $p = p_h = 10^{-12}$ should be used.
2. *The desired level of confidence is selected*. Compromise between the level of confidence and time of testing must be made. In order to reduce test time, the lowest real level of confidence is chosen. A compromise between the time of testing and the level of confidence is proportional to ln(1 – SLC).
3. Value *n is determined from* (*7*).
4. *Test time is calculated*. The time required to complete the test is n/R where R is the transmission rate.

The final equation is given using (7):

$$n = \frac{1}{\text{BER}}\left[-\ln\left(1-\text{SLC}\right) + \ln\left(\sum_{k=0}^{N} \frac{(n \cdot \text{BER})^k}{k!}\right)\right], \tag{8}$$

where n is the number of bits that need to be transferred, *BER* is the probability of incorrectly transmitted bits, N is the total number of detected errors and ln[.] is the natural logarithm. When there are no detected errors ($N = 0$), the second member in equation (8) is equal to zero and the solution of the equation is simplified a lot. When N is not zero, equation (8) can be solved empirically using a computer.

Now the equation (8) can be explained. Suppose we want to determine how many bits must be transferred without errors if we want to prove that error probability in the whole system is less than 10^{-10}, (BER < 10^{-10}), with the level of confidence 95%. In this example, $N = 0$, so that the second member in (8) is zero, and n is only dependent on SLC and BER. The result is n = (1/BER) [–ln (1 – 0.95)] ≈ 3/BER = $3 \cdot 10^{10}$.

This result shows a simple rule, that it is necessary to transmit three times the reciprocal of the specified BER without error, for the confidence level of 95%, that the specified BER is satisfied for a given system. Similar calculations show that n = 2.3/BER for 90% confidence level, or 4.6/BER for 99% confidence level, if there is no error in the system [1].

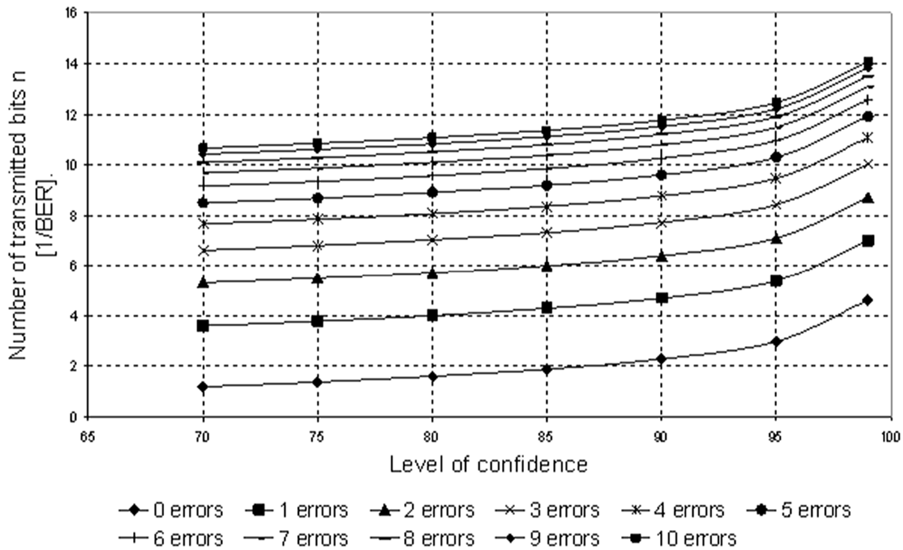

Fig. 3 – *Estimated number of transmitted bits, n, in function of the level of confidence between* 0 *and* 10 *error bits.*

Fig. 3 illustrates the relationship between the number of bits that must be transmitted and the confidence level for zero errors, one, two, and so on till ten errors. Results for commonly used confidence levels of 70% to 99% are shown in **Table 1**. In order to use the graph in Fig. 3, we select the desired level of confidence and through the point for this level of confidence we draw a vertical line until we cut the curve for a number of errors which were discovered during the test. From this intersection point, we pull a horizontal line to the left until we cut the vertical axis in order to determine the estimated number of bits n, which must be transmitted in the system for the desired level of confidence.

Table 1
Estimated number of transmitted bits, n, for the confidence level of 70% *to* 99%.

Number errors	$n = \ln(1 - \mathrm{SLC}) / \mathrm{BER}$						
	99%	95%	90%	85%	80%	75%	70%
0	$\frac{4{,}61}{\mathrm{BER}}$	$\frac{2{,}99}{\mathrm{BER}}$	$\frac{2{,}3}{\mathrm{BER}}$	$\frac{1{,}90}{\mathrm{BER}}$	$\frac{1{,}61}{\mathrm{BER}}$	$\frac{1{,}39}{\mathrm{BER}}$	$\frac{1{,}20}{\mathrm{BER}}$
1	$\frac{7{,}00}{\mathrm{BER}}$	$\frac{5{,}40}{\mathrm{BER}}$	$\frac{4{,}70}{\mathrm{BER}}$	$\frac{4{,}30}{\mathrm{BER}}$	$\frac{4{,}00}{\mathrm{BER}}$	$\frac{3{,}78}{\mathrm{BER}}$	$\frac{3{,}60}{\mathrm{BER}}$
2	$\frac{8{,}72}{\mathrm{BER}}$	$\frac{7{,}11}{\mathrm{BER}}$	$\frac{6{,}41}{\mathrm{BER}}$	$\frac{6{,}01}{\mathrm{BER}}$	$\frac{5{,}72}{\mathrm{BER}}$	$\frac{5{,}50}{\mathrm{BER}}$	$\frac{5{,}31}{\mathrm{BER}}$
3	$\frac{10{,}03}{\mathrm{BER}}$	$\frac{8{,}42}{\mathrm{BER}}$	$\frac{7{,}73}{\mathrm{BER}}$	$\frac{7{,}32}{\mathrm{BER}}$	$\frac{7{,}04}{\mathrm{BER}}$	$\frac{6{,}81}{\mathrm{BER}}$	$\frac{6{,}63}{\mathrm{BER}}$
4	$\frac{11{,}07}{\mathrm{BER}}$	$\frac{9{,}46}{\mathrm{BER}}$	$\frac{8{,}77}{\mathrm{BER}}$	$\frac{8{,}36}{\mathrm{BER}}$	$\frac{8{,}08}{\mathrm{BER}}$	$\frac{7{,}85}{\mathrm{BER}}$	$\frac{7{,}67}{\mathrm{BER}}$
5	$\frac{11{,}90}{\mathrm{BER}}$	$\frac{10{,}30}{\mathrm{BER}}$	$\frac{9{,}60}{\mathrm{BER}}$	$\frac{9{,}20}{\mathrm{BER}}$	$\frac{8{,}91}{\mathrm{BER}}$	$\frac{8{,}68}{\mathrm{BER}}$	$\frac{8{,}50}{\mathrm{BER}}$
6	$\frac{12{,}57}{\mathrm{BER}}$	$\frac{10{,}96}{\mathrm{BER}}$	$\frac{10{,}26}{\mathrm{BER}}$	$\frac{9{,}86}{\mathrm{BER}}$	$\frac{9{,}57}{\mathrm{BER}}$	$\frac{9{,}35}{\mathrm{BER}}$	$\frac{9{,}16}{\mathrm{BER}}$
7	$\frac{13{,}09}{\mathrm{BER}}$	$\frac{11{,}48}{\mathrm{BER}}$	$\frac{10{,}79}{\mathrm{BER}}$	$\frac{10{,}38}{\mathrm{BER}}$	$\frac{10{,}10}{\mathrm{BER}}$	$\frac{9{,}87}{\mathrm{BER}}$	$\frac{9{,}69}{\mathrm{BER}}$
8	$\frac{13{,}50}{\mathrm{BER}}$	$\frac{11{,}90}{\mathrm{BER}}$	$\frac{11{,}20}{\mathrm{BER}}$	$\frac{10{,}80}{\mathrm{BER}}$	$\frac{10{,}51}{\mathrm{BER}}$	$\frac{10{,}29}{\mathrm{BER}}$	$\frac{10{,}10}{\mathrm{BER}}$
9	$\frac{13{,}82}{\mathrm{BER}}$	$\frac{12{,}21}{\mathrm{BER}}$	$\frac{11{,}52}{\mathrm{BER}}$	$\frac{11{,}12}{\mathrm{BER}}$	$\frac{10{,}83}{\mathrm{BER}}$	$\frac{10{,}60}{\mathrm{BER}}$	$\frac{10{,}42}{\mathrm{BER}}$
10	$\frac{14{,}07}{\mathrm{BER}}$	$\frac{12{,}46}{\mathrm{BER}}$	$\frac{11{,}76}{\mathrm{BER}}$	$\frac{11{,}36}{\mathrm{BER}}$	$\frac{11{,}07}{\mathrm{BER}}$	$\frac{10{,}85}{\mathrm{BER}}$	$\frac{10{,}66}{\mathrm{BER}}$

5 Application of Procedure Level of Confidence in the Estimates of Bit Error Probability

In most of the telecommunication systems $P(e) \leq 10^{-10}$ is expected. Let us suppose that two systems should be tested, where the transmission bit rate is 2.5 Gbit/s (for the first one), and 10 Gbit/s (for the second one). First, we select $p_h = 10^{-10}$. Second, we would like to have the test that gives a 100% level of confidence in the desired specification, , but an infinite time of the test is required to fulfil this condition. Therefore, we choose level of confidence 99% and 90%. Then we solve n from the equation (8) using different values for N (N = 0, 1, 2, 3,..., N). The results are presented in **Table 2** for a confidence level of 99%, and in **Table 3** for a confidence level of 90%.

Table 2

Estimated number of required transmtted bits, n, and the required time of testing for SLC = 99% *and* $p_h = 10^{-10}$.

Number of error bits $(k) \leq N$ N	The required number of transferred bits n	Time of testing for a bit rate 2.5Gbit/s [s]	Time of testing for a bit rate 10Gbit/s [s]
0	$4.60 \cdot 10^{10}$	18.42	4.60
1	$7.00 \cdot 10^{10}$	28.01	7.00
2	$8.72 \cdot 10^{10}$	34.86	8.72
3	$1.00 \cdot 10^{11}$	40.13	10.03
4	$1.11 \cdot 10^{11}$	44.30	11.07
5	$1.19 \cdot 10^{11}$	47.61	11.90
6	$1.26 \cdot 10^{11}$	50.26	12.60
7	$1.31 \cdot 10^{11}$	52.37	13.10
8	$1.35 \cdot 10^{11}$	54.02	13.50
9	$1.38 \cdot 10^{11}$	55.30	13.82
10	$1.41 \cdot 10^{11}$	56.26	14.07

Table 3

Estimated number required transmitted bits, n, and the required time of testing for SLC = 90% *and* $p_h = 10^{-10}$.

Number of error bits $(k) \leq N$ N	The required number of transferred bits n	Time of testing for a bit rate 2.5Gbit/s [s]	Time of testing for a bit rate 10Gbit/s [s]
0	$2.30 \cdot 10^{10}$	9.21	2.30
1	$4.70 \cdot 10^{10}$	18.80	4.70
2	$6.41 \cdot 10^{10}$	25.65	6.41
3	$7.73 \cdot 10^{10}$	30.92	7.73
4	$8.77 \cdot 10^{10}$	35.08	8.77
5	$9.60 \cdot 10^{10}$	38.40	9.60
6	$1.03 \cdot 10^{11}$	41.05	10.26
7	$1.08 \cdot 10^{11}$	43.16	10.79
8	$1.12 \cdot 10^{11}$	44.81	11.20
9	$1.15 \cdot 10^{11}$	46.09	11.52
10	$1.18 \cdot 10^{11}$	47.05	11.76

Table 2 shows that, if no erroneous bits are detected during 18.42 s in a 2.5 Gbit/s system, we get a confidence level of 99% that the bit error probability is $P(e) \leq 10^{-10}$, with transfer of at least $n = 4.60 \cdot 10^{10}$ bits. If one bit error occurs in 28.01 s of testing, or two incorrect bits occur 34.86 s, the result is the same: the level of confidence is 99% that bit error probability is $P(e) \leq 10^{-10}$, with a minimum number of transmitted bits $n = 7.0 \cdot 10^{10}$ and $n = 8.72 \cdot 10^{10}$ bits, respectively.

5 Reducing Time of Tests

Tests in which we require a high level of confidence and/or small *BER*, can last a long time, especially for systems with low-rate data transfer. The time required to complete the test is expressed by the following formula:

$$VT[s] = \frac{n}{R}, \tag{9}$$

where: *VT* is time (given in seconds) needed to complete the test, *n* is the number of transferred bits and *R* is bit data rate in the system.

Let us imagine a test with 99% level of confidence in the obtained results, and with BER = 10^{-12} for a system, which has a bit rate of 622 Mbps. From **Table 1**, we can see that the required number of bits is $4.61 \cdot 10^{12}$ for zero errors. For 622 Mbps, the time of testing will be 4.61·1012 bits / 622·106 bits/s = 7411 s, which is slightly more than two hours. Two hours is usually too long time for a practical test, but what can be done to reduce the testing time?

One common method is to shorten the time of testing, which involves deliberate reduction of the signal to noise ratio (SNR) in the system. This gives a higher level of bit error and faster measurement with degraded BER in order to obtain the required results, [6]. If we know the relationship between SNR and BER, then the results for degraded BER can be extrapolated to estimate BER. The application of this method is based on the assumption that the thermal (*Gaussian*) noise is dominant cause of errors in the system at the receiver input.

The relationship between SNR and BER can be derived using *Gaussian* statistics and is documented in many books on communications [7]. Although there is no known closed form solution for the relationship between SNR and BER, the results can be obtained by numerical integration. One convenient method for calculating this relationship is to use *Microsoft Excel*™ standard normal distribution, NORMSDIST[.]. Using this function, the ratio of SNR and BER can be calculated as:

$$\text{BER} = 1 - \text{NORMSDIST}(\text{SNR} / 2) . \tag{10}$$

Besides this, ratio SNR and BER can also be calculated using equation (11), which uses the *Q* factor (optical systems [8]), [9]:

$$\text{BER} = \int_Q^{\infty} \text{Gauss}(x)\,\mathrm{d}x = \frac{1}{\sqrt{2\pi}} \int_Q^{\infty} \mathrm{e}^{\frac{-x^2}{2}}\,\mathrm{d}x = \\ = \frac{1}{2}\operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{1}{\sqrt{2\pi}} \frac{\exp\left(\frac{-Q^2}{2}\right)}{Q}, \tag{11}$$

where Q^2, according to the literature [9], depends on the level of noise at 1 and 0. We get two values: $Q^2 = 2$ SNR when the noise level at 1 is much higher than the noise level at 0, and $Q^2 =$ SNR when the noise level at 1 is the same as at 0.

Fig. 4 presents the value of BER in the function of SNR for the two methods of calculation using equation (10) and equation (11). To illustrate this method of accelerated testing, we start from the example, presented at the beginning of the section. In this example, testing in the case of BER = 10^{-12} for 99% confidence level in the transmission system of 622 Mbps would take more than two hours.

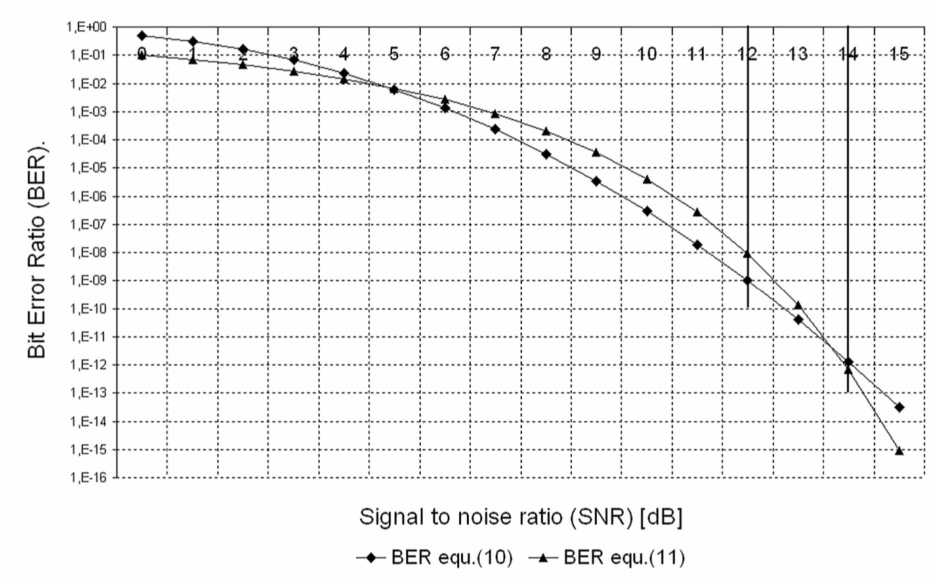

Fig. 4 – *The relationship between* BER *and* SNR.

From Fig. 4, we see that the BER of 10^{-12} corresponds to SNR of about 14. The communication system under test may be a signal channel between the transmitter and receiver, in which the attenuator is inserted. Since the signal is attenuated before the receiver input, then, on the assumption that the dominant source of noise at the receiver input remained the same, we weakened the signal, not noise. Therefore, SNR will be reduced by the same amount as the signal. (It is important to ensure that the signal is not weakened below noise level for a given channel). For this example, SNR is reduced from 14 to 12 by inserting 0.67 dB attenuation. From Fig. 4 follows that reducion of SNR to 12 corresponds to changing BER on 10^{-8}, according to equation (11), and on 10^{-9}, according to equation (10). For a confidence level of 99% in the case of BER = 10^{-9}, it should be transferred $4.61 \cdot 10^{9}$ bits (**Table 1**). The duration of testing is 7.41 s, according to equation (9). This duration is 1000 times smaller than the original time of testing. So, if we have no errors during testing which lasts 7.41 s when we are using the attenuator, we can say that BER is 10^{-9}. Then, with extrapolation, when we remove the attenuation, we should get a BER of 10^{-12}.

Shortening of testing time by reduction of the SNR and the use of extrapolation decrease the level of confidence in the obtained results. Decreasing the level of confidence becomes more important when the extrapolation distance becomes greater. To show this effect, the test will be considered where BER is decreased by a factor of 100 as the result of *SNR* reduction. If the test for the reduced SNR is performed for the 99% confidence level with zero errors, then it can be expected that, if the test is repeated 100 times, 99 tests are with zero errors and one test is with one error. If we now combine all 100 received repeated tests, we get 100 times more bits with error. Extrapolation of results of 100 repeated tests on the original does not reduce the level of BER, gives one bit error in 1/BER bits, or it is $n \cdot$BER = 1.0. Using equation (8) for $N = 0$ or $n \cdot$BER = 1, we obtain

$$\begin{gathered} n \cdot \mathrm{BER} = 1 = -\ln\left(1 - \mathrm{SLC}\right), \\ \mathrm{SLC} = 1 - \mathrm{e}^{-1} \approx 0.63. \end{gathered} \tag{12}$$

We get the appropriate level of confidence that is approximately only 63%, small enough to be out the graphic on Fig. 3. This value is far from 99%, level of confidence from which we started.

On the basis of the examples, the SNR should be reduced as little as possible to achieve reasonable testing time. It must be understood that the extrapolation reduces the level of confidence. Also, measurements and calculations must be made with special precision. Errors those occur due to rounding, the measurement tolerance, etc., will be magnified when we extrapolate the results.

6 Conclusion

Bit Error Ratio (BER) for digital communication systems is an important measure, which is used to quantify the integrity of data transmitted through the system. Finite testing time provides that estimation of the probability erroneous bits will be received. The quality of estimation is enhanced when the test time increases, and this quality can be quantified using methods of statistical level of confidence. Ideas for reducing the test time have been already published, but these ideas should be used with caution, because their using can significantly reduce the level of confidence.

The proposed procedures for determining the required number of transferred bits n, based on the selected value of bit error probability and statistical level of confidence, are presented in use with optical systems. These optical systems, with the rate 2.5 Gbit/s and 10 Gbit/s, are developed by IRITEL.

For the calculation of the required number of transmitted bits, n, we took the value for $np = 10$ (the condition $np > 1$ is fulfilled), and for all calculations in the paper programs MATLAB and MATHEMATICA are used.

6 Acknowledgement

This work is partially supported by the Ministry of Science and Technological Development of the Republic of Serbia within the Project TR32007.

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