A Solution of Two-Dimensional Magnetohydrodynamic Flow Using the Finite Volume Method

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Abstract: This paper presents the two dimensional numerical modeling of the coupling electromagnetic-hydrodynamic phenomena in a conduction MHD pump using the Finite volume Method. Magnetohydrodynamic problems are, thus, interdisciplinary and coupled, since the effect of the velocity field appears in the magnetic transport equations, and the interaction between the electric current and the magnetic field appears in the momentum transport equations. The resolution of the Maxwell's and Navier Stokes equations is obtained by introducing the magnetic vector potential A, the vorticity ζ and the stream function ψ. The flux density, the electromagnetic force, and the velocity are graphically presented. Also, the simulation results agree with those obtained by Ansys Workbench Fluent software.

Keywords: Magnetohydrodynamic, Conduction pump, velocity, Maxwell's equations, Navier-Stokes equations, Electrode, Finite Volume Method, Ansys Fluent.

1 Introduction

Magnetohydrodynamics (MHD) is the study of the motion of electrically conducting fluids in the presence of magnetic fields. Effects from such interactions can be observed in liquids and gases. A number of researches have investigated the flow of an electrically conducting fluid through channels because of its important applications in MHD generators, pumps, accelerators, flow meters and blood flow measurements. The pumping of liquid metal may use an electromagnetic device, which induces eddy currents in the metal. These induced currents and their associated magnetic field generate the Lorentz force whose effect can be actually the pumping of the liquid metal [1 – 8].

The advantage of these pumps which ensure the energy transformation is the absence of moving parts.

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In the MHD pump, the pumping forces are represented by the Lorentz forces induced by the interaction between the applied electrical currents and the magnetic fields [3, 5]. Therefore, the basic concept of the DC MHD pump is to apply electrical currents across a channel filled with electrically conducting liquids and orthogonal magnetic fields via electromagnetic circuit [9, 10].

In the previous work [7] we have studied the electromagnetic phenomena in a MHD conduction pump.

The purpose of this paper is to determine the velocity profile in the channel of Magnetohydrodynamic conduction pump using the code developed with the finite volume method in Matlab. Also, the comparison of the obtained results with Ansys Fluent software is presented.

2 Governing Equations

2.1 Electromagnetic problem

The scheme of the MHD pump is shown in the Fig. 1. The governing equations for the fluid flow are based on the Lorentz forces and steady state of the electric properties of the fluid. Then, the Lorentz forces can be written as follows [11, 14]:

$$F = (J_{ind} + J_a) \nu B . \tag{1}$$

The Maxwell's equations applied to the MHD pump will give rise to the following equation:

$$\text{rot}\left(\frac{1}{\mu}\text{rot}\,A\right) = J_{ex} + J_a + \sigma(V \wedge B) , \tag{2}$$

where A the magnetic vector potential, B is the magnetic field obtained using the electromagnetic circuit, J_{ind} is induced current density, J_a the current density applied to the electrode, σ is the electrical conductivity, μ the magnetic permeability, V is the velocity of the fluid and J_{ex} the excitation current density in the coils.

For the calculation reported in the following, mercury is considered as electroconductive liquid. For 2D plane model in Cartesian coordinates (x, y, z) where the vectors A, J_{ex} and J_a are along the z-axis. The equation (2) developed in Cartesian coordinates becomes:

$$\nu\nu - \frac{1}{\mu}\left(\frac{\nu^2 A}{\nu x^2} + \frac{\partial^2 A}{\nu y^2}\right) = J_{ex} + J_a + \sigma\left(-V_x\frac{\partial A}{\partial x} - V_y\frac{\partial A}{\partial y}\right). \tag{3}$$

The channel dimensions and the properties of the mercury are given respectively in **Tables 1** and **2**.
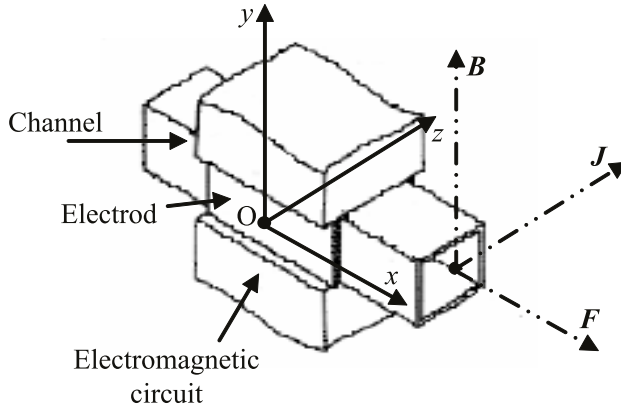
Fig. 1 – *Scheme of a DC MHD pump* [3].

Table 1
Channel's dimensions.

Parameter	Values
Channel's length (L)	0.2 m
Channel's width (w)	0.2 m

Table 2
Fluid properties.

Parameter	Mercury solution
Density ρ	$13.6 \cdot 10^3$ (kg/m^3)
Electrical conductivity σ	$1.06 \cdot 10^6$ (Sm^{-1})
Relative permeability	1
Viscosity μ	$0.11 \cdot 10^{-6}$ (m^2/s)

2.2 The hydrodynamic problem

The MHD transient flow of an incompressible, viscous and electrically conducting liquid is governed by the Navier-Stokes equations [9]:

$$\frac{\partial V}{\partial t} + (\nabla \cdot V)V = -\frac{1}{\rho}\nabla p + \nu \Delta V + \frac{F}{\rho},$$

$$\mathrm{div}\, V = 0,$$

(4)

where p is the pressure of the fluid, ν is the kinemactic viscosity of the fluid, F is the electromagnetic force and ρ the fluid density [12, 14].

In Cartesian coordinates these equations becomes [16]:

$$\frac{\partial Vx}{\partial t} + Vx\frac{\partial Vx}{\partial x} + Vy\frac{\partial Vx}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 Vx}{\partial x^2} + \frac{\partial^2 Vx}{\partial y^2}\right] + \frac{1}{\rho}Fx$$

$$\frac{\partial Vy}{\partial t} + Vx\frac{\partial Vy}{\partial x} + Vy\frac{\partial Vy}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 Vy}{\partial x^2} + \frac{\partial^2 Vy}{\partial y^2}\right] + \frac{1}{\rho}Fy \qquad (5)$$

$$\frac{\partial Vx}{\partial x} + \frac{\partial Vy}{\partial y} = 0,$$

The difficulty is that in the previous equations there are two unknown: the pressure and the velocity. The elimination of pressure from the equations leads to a vorticity-stream function which is one of the most popular methods for solving the 2-D incompressible Navier-Stokes equation

$$V = \text{rot}\,\psi, \quad \zeta = \text{rot}\,V, \qquad (6)$$

$$\frac{\partial \psi}{\partial y} = Vx, \quad \frac{\partial \psi}{\partial x} = -Vy \qquad (7)$$

In the present situation, ζ has only one component in the *z* direction

$$\zeta = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}, \qquad (8)$$

where V_x and V_y the components of the velocity V.

Using these new dependent variables, the two momentum equations can be combined (there by eliminating pressure) to give:

$$\frac{\partial \zeta}{\partial t} + Vy\frac{\partial \zeta}{\partial y} + Vx\frac{\partial \zeta}{\partial x} = \nu\left[\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2}\right] + \frac{1}{\rho}\left(\frac{\partial Fx}{\partial y} + \frac{\partial Fy}{\partial y}\right). \qquad (9)$$

After substituting (7) into (9) we obtain an equation involving the new dependant variables ζ and ψ such as:

$$-\zeta = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}. \qquad (10)$$

3   Numerical Method

There are several methods for the determination of the electromagnetic fields and the velocity; the choice of the method depends on the type of problem [10, 13].

In our work, we thus choose the finite volume method; its principle consists on subdividing the field of study (Ω) in a number of elements. Each element

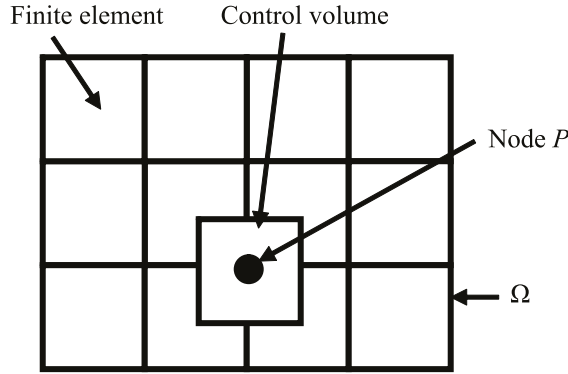contains four nodes of the grid. A finite volume surrounds each node of the grid, Fig. 2 [15].

Finite element   Control volume

Node P

Ω

Fig. 2 – *Grid of the domain.*

The method consists of discretizing the differential equations by integration on finite volumes surrounding the nodes of the grid. In this method, each principal node P is surrounded by four nodes N, S, E and W located respectively at North, South, East and West (Fig. 3).
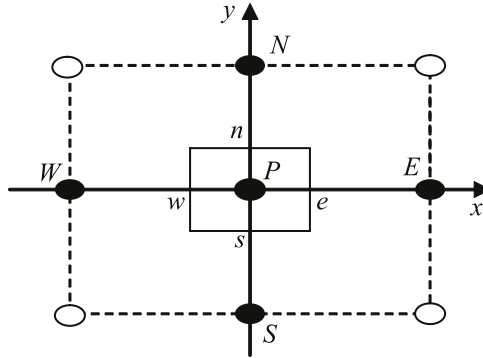
Fig. 3 – *Description of the finite volume* [11].

We integrate the electromagnetic equation in the finite volume method delimited by the neighboring nodes E, W, N and S as:

$$\int_{w}^{e}\int_{s}^{n}\left[\frac{1}{\mu}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)\right]\mathrm{d}x\,\mathrm{d}y = \int_{w}^{e}\int_{s}^{n}\left(J_{ex}+J_a+\sigma V_x\frac{\partial A}{\partial x}\right)\mathrm{d}x\,\mathrm{d}y. \quad (11)$$

After integration, the final algebraic equation will be:

$$a_p A_p = a_e A_e + a_w A_w + a_n A_n + a_s A_s + d_p \quad (12)$$

with:

$$a_e = \frac{\Delta y}{\mu_e (\delta x)_e}, \quad a_w = \frac{\Delta y}{\mu_w (\delta x)_w}, \quad a_n = \frac{\Delta x}{\mu_n (\delta y)_n},$$

$$a_s = \frac{\Delta x}{\mu_s (\delta y)_s}; \quad a_p = a_e + a_w + a_n + a_s.$$

The same procedure is applied equations (9) and (10) in order to obtain the velocity of the fluid in the channel of the MHD pump.

The resolution of the electromagnetic and the hydrodynamic equations allows the calculation of the magnetic vector potential A and then the magnetic flux density B, the electromagnetic force F and the velocity V in the channel of the conduction MHD pump.

4   Application and Results

The transverse section of the MHD pump used in this study is given in Fig. 4.
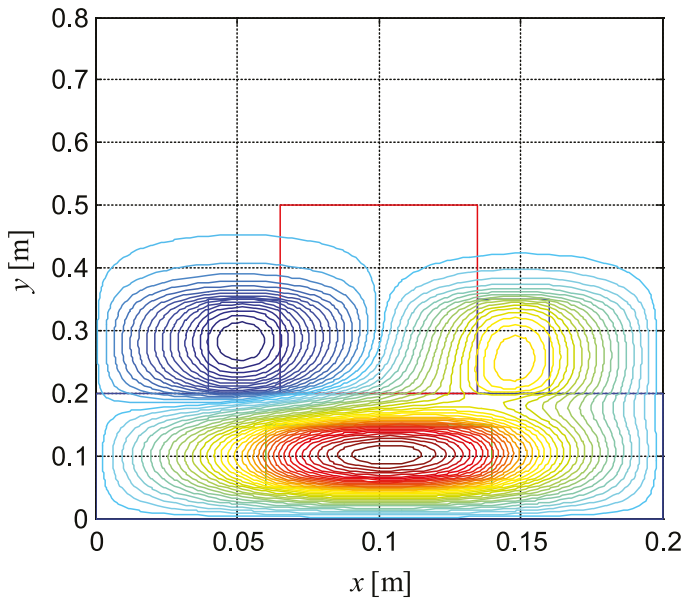
The Figs. 5 and 6 represent respectively the equipotential lines and the distribution of the magnetic vector potential in the MHD pump.

The Fig. 7 represents the magnetic flux density in the channel. It is shown that, the magnetic induction reaches its maximum value at the inductor.
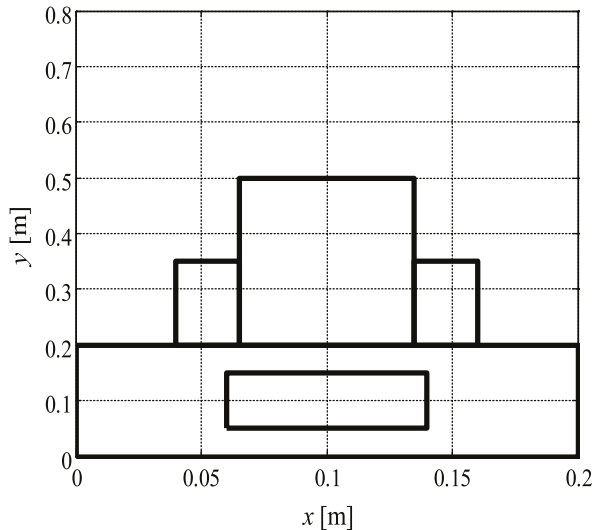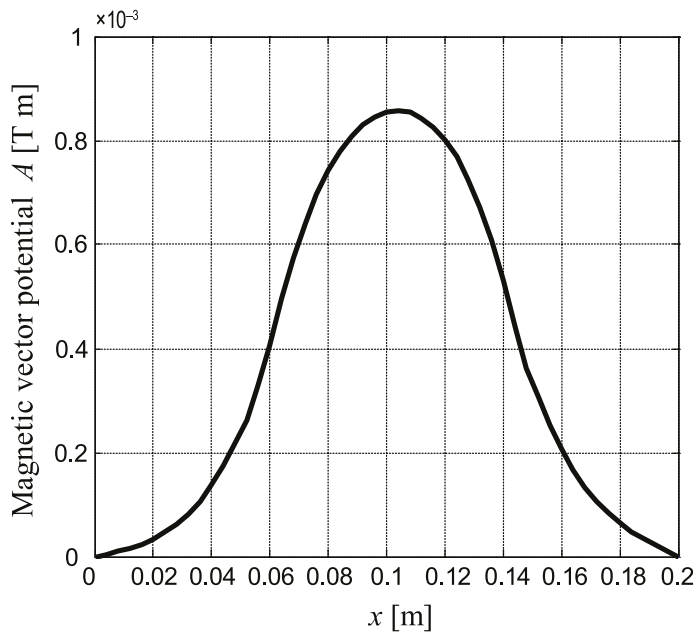
Fig. 4 – *A conduction MHD pump configuration.*

Fig. 5 – *Equipotential lines in a DC MHD pump.*

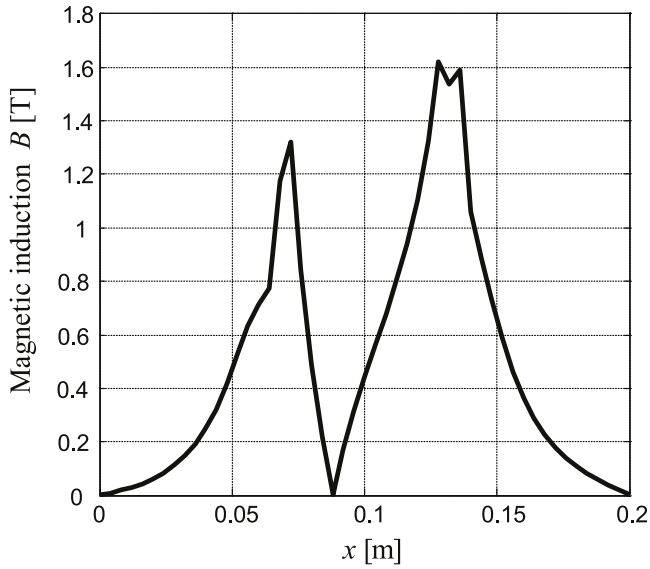**Fig. 6** – *Magnetic vector potential in a DC MHD pump.*

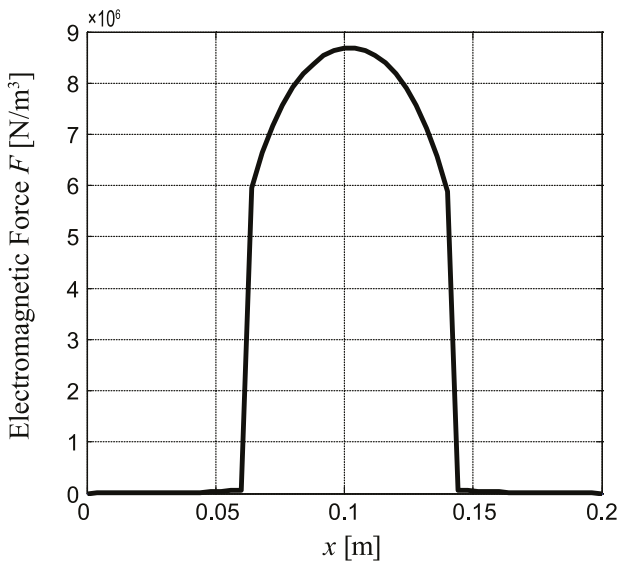**Fig. 7** – *Magnetic induction in the MHD pump.*

Fig. 8 – *Electromagnetic force in the channel of the MHD pump.*

The Fig. 8 represents the electromagnetic force in the channel. It is noted that the maximum value of the force is in the middle of the channel of the MHD pump.
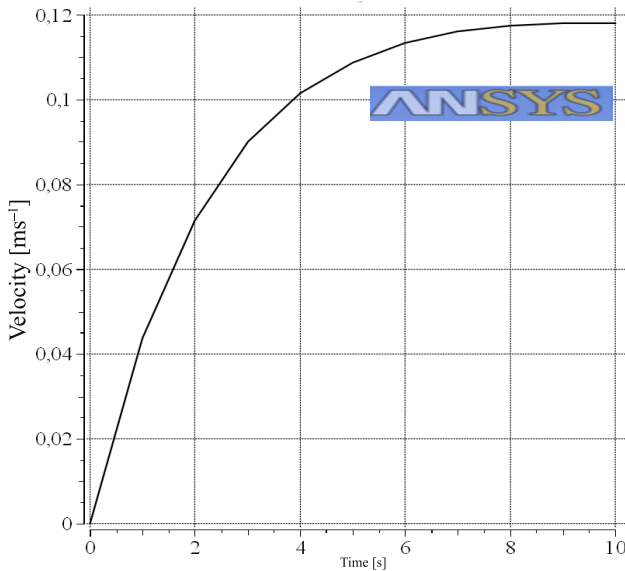
5   Validation of the Results

The Software ANSYS is a tool for the simulation of the physical problems using finite element method. It allows the analysis of different problems such as electromagnetic, hydrodynamic and thermal ones.
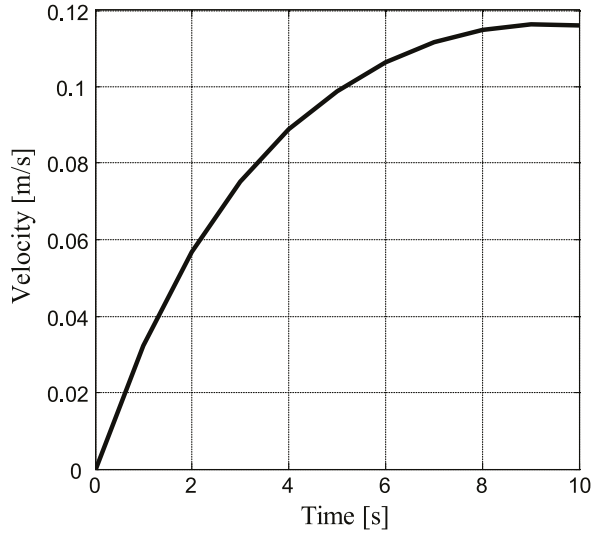
(a)

(b)

Fig. 9 – (a) *Using finite volume method*; (b) *Using Ansys*.

The Fig. 9a represents the velocity in the channel of the MHD pump. It is shown that the velocity of the fluid flow passes by a transitory mode then is stabilized like all electric machines and the steady state is obtained approximately after six seconds. The results obtained are almost identical qualitatively to those obtained by [6, 14].

The results from the ANSYS Fluent are used to represent the velocity in the channel of the DC MHD pump, Fig. 9b.

6   Conclusion

In this paper we have studied the magnetohydrodynamic problem using 2D finite volume method taking into account the movement of the fluid.

Various characteristics such as the distribution of the magnetic vector potential, the magnetic flux density, the electromagnetic force and the velocity are given. The results of the velocity obtained are almost identical qualitatively to those obtained by [6] and [14].

According to the results of the velocity obtained in the channel of a conduction MHD pump, we note that the results simulated by MATLAB are the same to those simulated by ANSYS Fluent.

7   References

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