SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 4, No. 2, November 2007, 133-145

Model Based Fault Detection and Diagnosis Using Structured Residual Approach in a Multi-Input Multi-Output System

A. Asokan[1], D. Sivakumar[2]

Abstract: Fault detection and isolation (FDI) is a task to deduce from observed variable of the system if any component is faulty, to locate the faulty components and also to estimate the fault magnitude present in the system. This paper provides a systematic method of fault diagnosis to detect leak in the three-tank process. The proposed scheme makes use of structured residual approach for detection, isolation and estimation of faults acting on the process [1]. This technique includes residual generation and residual evaluation. A literature review showed that the conventional fault diagnosis methods like the ordinary Chi-square (Ψ^2) test method, generalized likelihood ratio test have limitations such as the "false alarm" problem. From the results it is inferred that the proposed FDI scheme diagnoses better when compared to other conventional methods.

Keywords: Fault detection and diagnosis, Residual generation, Structured residual approach

1 Introduction

Faults can occur either in the processing equipment (leak in a tank) or in the auxiliary equipment like sensors and actuators. These can result in degradation of closed loop performance and also have an impact on safety, productivity and plant economy. Therefore, it is necessary to assess the process performance and diagnose the cause of performance degradation using model based fault detection and identification. Since the early 1970s fault detection and isolation have attracted increasing research attention. This lead to the development of various approaches through use of redundant hardware, Kalman filter and observer [2], parity equations and directional and structured residual [3, 8, 12].

Among the various FDI schemes, the structured residual approach (SRA) proposed by J. Gertler, M. Staroswiecki and M. Shen [4] is powerful in isolating faults. SRA proposed by Gertler [10, 11] is further simplified for a multi input

[1]Department of EIE, Annamalai University, Tamilnadu, India; E-mail: asokan_me@yahoo.co.in
[2]Department of EIE, Annamalai University, Tamilnadu, India

multi output system and it is considered for this paper for fault detection and isolation for a three tank system. The SRA involves two steps i) generation of Primary Residual Vector (PRV) for fault detection and ii) transformation of PRV into structured residual vector (SRV) for fault isolation.

The implementation procedure of the proposed FDI scheme is illustrated in Fig. 1. The controller in the system is used to maintain the process variable at its set point. When there is a fault in the process, its output differs with model output. This difference is termed as residual. By simply monitoring the residuals one can say that something is going wrong. But it is not possible to identify the location of the fault. So the residual has to be processed to enhance isolation. In this paper the structured residual approach is applied to a MIMO system to enhance fault isolation.

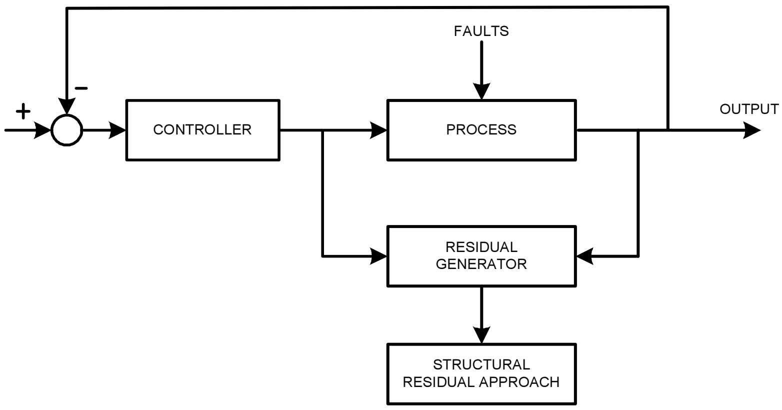

Fig. 1 – *Block diagram representation of proposed FDI scheme.*

This paper is organized as follows. In section 2 the system under study which is the three-tank system is described. In section 3, the identification of un-measured disturbance variables (faults) using residual approach as reported in literature is explained. The proposed scheme to identify and estimate the un-measured disturbance acting on the process is presented in section 4. In section 5 the simulation results are discussed. Finally the conclusions are drawn and scope of further work is provided in section 6.

2 System Descriptions

The three-tank system considered for study [6] is shown in Fig. 2. The controlled variables are the level of the tank1 (h_1) and level of the tank3 (h_3). In flow of tank1 (fin_1) and in flow of tank3 (fin_3) are chosen as manipulated variables to control the level of the tank1 and tank3. The unmeasured outflow of that is leak of tank1, tank2 and tank3 have been considered as fault variables.

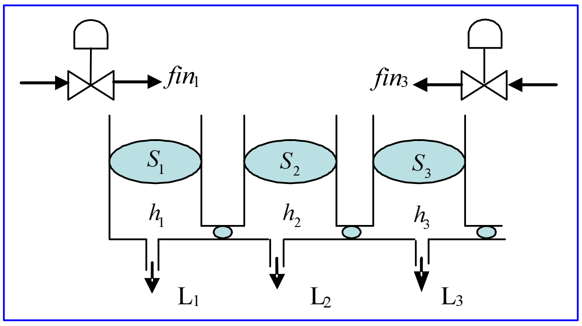

Fig. 2 – *Three Tank System.*

The material balance equation for the above three-tank system is given by

$$\frac{\mathrm{d}h_1}{\mathrm{d}t} = \frac{fin_1}{S_1} - \frac{\mathrm{Az}_1}{S_1}\sqrt{2g(h_1 - h_2)} - \frac{L_1}{S_1} \tag{1}$$

$$\frac{\mathrm{d}h_2}{\mathrm{d}t} = \frac{\mathrm{Az}_1}{S_2}\sqrt{2g(h_1 - h_2)} - \frac{\mathrm{Az}_3}{S_2}\sqrt{2g(h_1 - h_3)} - \frac{L_2}{S_2} \tag{2}$$

$$\frac{\mathrm{d}h_3}{\mathrm{d}t} = \frac{fin_3}{S_3} + \frac{\mathrm{Az}_3}{S_3}\sqrt{2g(h_2 - h_3)} - \frac{\mathrm{Az}_2}{S_3}\sqrt{2gh_3} - \frac{L_3}{S_3} \tag{3}$$

The steady state operating data of the Three-tank system is given in **Table 1**.

Table 1

Steady state operating data.

h_1, h_2, h_3 [m]	0.7, 0.5, 0.3
fin_1, fin_3 [ml/s]	100
Product of outflow coefficient and cross section of the connection pipe (Az_1, Az_2, Az_3) [m^2]	$2.25*10^{-5}$, $3.0571*10^{-5}$, $2.307*10^{-5}$
Area of tank (S_1 - S_3) [m^2]	0.0154
L_1, L_2, L_3 [ml/s]	0
Acceleration due to gravity, g [m/s^2]	9.81

3 Fault [leak] Detection Using Residual Generator

Residuals are generated from the observable variable of the monitored plant, that is, from the command values of the controlled inputs and the outputs [5]. Ideally, the residuals should only be affected by the faults. However, the presence of disturbances, noise and modeling errors also causes the residuals to become nonzero and thus interferes with the detection of faults. Therefore the residual generator needs to be designed so that it is maximally unaffected by these nuisance inputs, which means that it is robust in the face of disturbance, noise and model errors. Structured residual are so designed that each residual responds to a different subset of faults and insensitive to the others. When a particular fault occurs, some of the residuals do respond and others do not. Then the pattern of the response set, the fault signature or fault code, is characteristic of the particular fault.

Example for fault code:

$$\begin{array}{c c} & \begin{array}{ccc} R_1 & R_2 & R_3 \end{array} \\ \begin{array}{c} L_1 \\ L_2 \\ L_3 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

The above fault code implies that fault L_1 affects only residual R_1 like L_2 affects R_2 and L_3 affects R_3. In order to perform detection and isolation of set of faults, structured residuals can be used. The so called signature code describes the subset of residuals which react to each fault. Since the levels of all three-tank are assumed to be measurable, there will be three residuals corresponding to each of the three tanks When there is a fault, all the three residuals get affected. By simply monitoring the residuals it is possible to predict the change in behavior of the system from normal. But it is not possible to identify the location of the fault. So the residual has to be transformed to enhance isolation.

Let the plant output is given by

$$Y_P(s) = G(s)U(s) + G_F(s)L(s), \tag{4}$$

where: $G(s)$ – transfer function under normal conditions and
$G_F(s)$ – fault transfer function.

Let the output of the model be given by

$$Y_M(s) = G(s)U(s), \tag{5}$$

where

$$U(s) = \begin{bmatrix} fin_1(s) \\ 0 \\ fin_3(s) \end{bmatrix} \text{ and } L(s) = \begin{bmatrix} L_1(s) \\ L_2(s) \\ L_3(s) \end{bmatrix}.$$

Residual R(s) is defined as difference between process output and model output.

$$R(s) = Y_P(s) - Y_M(s) . \tag{6}$$

Substituting the expressions for $Y_P(s)$ and $Y_M(s)$ in equation (6)

$$R(s) = G_F(s)\ L\ (s),$$

$$\begin{bmatrix} R_1(s) \\ R_2(s) \\ R_3(s) \end{bmatrix} = \begin{bmatrix} G_{F11} & G_{F12} & G_{F13} \\ G_{F21} & G_{F22} & G_{F23} \\ G_{F31} & G_{F32} & G_{F33} \end{bmatrix} \begin{bmatrix} L_1(s) \\ L_2(s) \\ L_3(s) \end{bmatrix}, \tag{7}$$

$$R_1(s) = G_{F11}(s)L_1(s) + G_{F12}(s)L_2(s) + G_{F13}(s)L_3(s) , \tag{8}$$

$$R_2(s) = G_{F21}(s)L_1(s) + G_{F22}(s)L_2(s) + G_{F23}(s)L_3(s) , \tag{9}$$

$$R_3(s) = G_{F31}(s)L_1(s) + G_{F32}(s)L_2(s) + G_{F33}(s)L_3(s) . \tag{10}$$

The above equation is valid only in the absence of plant model mismatch and in the absence of state and measurement noise. From the above equations, it is evident that the presence of fault will affect all the three residuals. It is difficult to identify the location of fault by monitoring the residuals. Therefore in order to enhance the fault isolation it is required to transform the residuals. The design of transformation matrix is discussed in the subsequent section.

4 Design of Transformation Matrix

To transform raw residual $R(s)$ into structured form $R_t(s)$, multiply $R(s)$ with weighting matrix $W(s)$

$$R_t(s) = W(s)R(s) . \tag{11}$$

Weighting matrix is chosen as

$$W(s) = Z(s)G_F^{-1}(s) . \tag{12}$$

Substituting $W(s)$ in expression (11)

$$R_t(s) = [Z(s)G_F^{-1}(s)][R(s)],$$

$$R_t(s) = [Z(s)G_F^{-1}(s)][G_F(s)L(s)], \tag{13}$$

$$R_t(s) = Z(s)I\,L(s),$$

where I is the identity matrix,

$$R_t(s) = Z(s)L(s) . \tag{14}$$

If $Z(s)$ is the diagonal matrix then

$$R_{1t}(s) = Z_{11}(s)L_1(s), \tag{15}$$

$$R_{2t}(s) = Z_{22}(s)L_2(s), \tag{16}$$

$$R_{3t}(s) = Z_{33}(s)L_3(s). \tag{17}$$

It is inferred that the first element of $R_t(s)$ that is $R_{1t}(s)$ affected only if there is a leak in the first tank. The second element of $R_t(s)$ that is $R_{2t}(s)$ affected only if there is a leak in the second tank. Like that the third element of $R_t(s)$ that is $R_3(s)$ affected only if there is a leak in the third tank.

The transformation matrix $W(s)$ is

$$W(s) = \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix}, \tag{18}$$

where:

$$W_{11} = 1, \quad W_{12} = \frac{-0.0011}{0.0154s + 0.00011}, \quad W_{13} = 0,$$

$$W_{21} = \frac{-0.0011}{0.0154s + 0.00011}, \quad W_{22} = 1, \quad W_{23} = \frac{-0.0015}{0.0154s + 0.00011},$$

$$W_{31} = 0, \quad W_{32} = \frac{-0.0015}{0.0154s + 0.00011}, \quad W_{33} = 1.$$

The user specified residual specification matrix $Z(s)$ is given by

$$Z(s) = \begin{bmatrix} Z_{11} & 0 & 0 \\ 0 & Z_{22} & 0 \\ 0 & 0 & Z_{33} \end{bmatrix}, \tag{19}$$

where:

$$Z_{11} = \frac{1}{0.0154s + 0.000112}, \; Z_{22} = \frac{1}{0.0154s + 0.00026}, \; Z_{33} = \frac{1}{0.0154s + 0.000193}.$$

5 Simulation Results

The proposed FDI scheme has been implemented on a three-tank system and its performance is observed. The controlled variables are the level of tank1 (h_1) and tank3 (h_3). Inflow of tank1 (fin_1) and tank3 (fin_3) are chosen as manipulated inputs. Outflow of tank1 (L_1), tank2 (L_2) and tank3 (L_3) are considered as leak variables.

The synthesis method [8] is used for the design of PI controller. The PI controllers are designed so that the closed loop process behaves like a first order system with unity gain and time constant same as the open loop time constant. The resulting parameter for controlling the height of tank1 using the inflow of tank1 is given by $K_c = 2.54 \cdot 10^{-4}$[ml/ s/ m] and $T_i = 222$[s] and that of tank3 using the inflow of tank3 is given by $K_c = 7.69 \cdot 10^{-4}$[ml/ s/ m] and $T_i = 200$[s].

The process is simulated using the non-linear first principles model, whereas the FDI is based on the time invariant linearized model (Transfer function model). The closed loop behavior of the process when a leak of magnitude 50[ml/s] introduced at time $t = 3000$[s] in tank1 is shown in Fig. 3. The behavior of the residuals is shown in Fig. 4. From the Fig. 4 one can infer the presence of leak in tank1 affect all the three residuals. By simply monitoring either the Process output or the residual it is not possible to identify the location of the fault. The Structured residual output is shown in Figs. 5, 6 and 7. From these figures one can conclude that there is leak only in the first tank.

Closed loop response of the System when leak occurs in all the three-tanks is shown in the Fig. 8 that is, a leak of magnitude 50[ml/s] given in tank1 at $t = 2750$[s], leak of magnitude 50[ml/s] given in tank2 at $t = 4000$[s] and leak of magnitude 100[ml/s] given in tank3 at $t = 6000$[s]. It is observed that the levels of the tank are maintained even though the fault occurs in the process. So simply monitoring the process output it is not possible to detect the fault. The behavior of the residuals is shown in Fig. 9. With the residual one cannot find the location of fault.

The Structured residual outputs are shown in Figs. 10, 11 and 12. From these figures one can conclude that there is leak in the all the three tanks. The Structured residual approach is tested for modeling errors that is 10% deviation in time constant is considered. The closed loop behavior of the process when a leak of magnitude 50[ml/s] introduced at time $t = 6000$[s] in tank3 is shown in Fig. 13. The behavior of the residuals is shown in Fig. 14. The Structured residual outputs are shown in Figs. 15, 16 and 17. From these figures one can conclude that there is leak in the third tank only.

Structured residual approach is also tested for another set of controller parameters. The PI controller settings are obtained so that the closed loop process behaves like a first order system with unity gain and time constant 10% less than the open loop time constant. The closed loop behavior of the process when a leak of magnitude 100[ml/s] introduced at time $t = 6000$[s] in tank3 under the new settings is shown in Fig. 18. The behavior of the residuals is shown in Fig. 19. The Structured residual outputs are shown in Figs. 20, 21 and 22. From these figures one can conclude that there is leak in the third tank only.

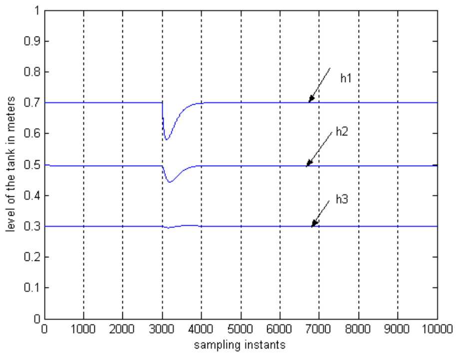

Fig. 3 – *Closed loop response of the system when a leak of magnitude* 50 [ml/s] *occurs in tank1 at* t = 3000[s].

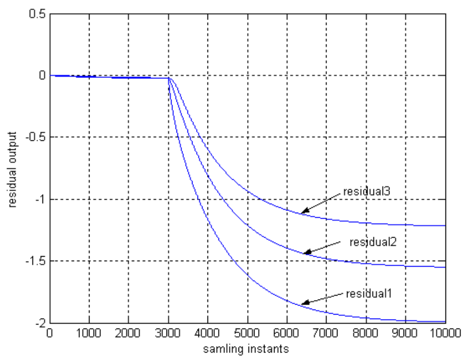

Fig. 4 – *Evolution of residuals when step change in leak of tank1 of magnitude* 50 [ml/s] *introduced at* t = 3000[s].

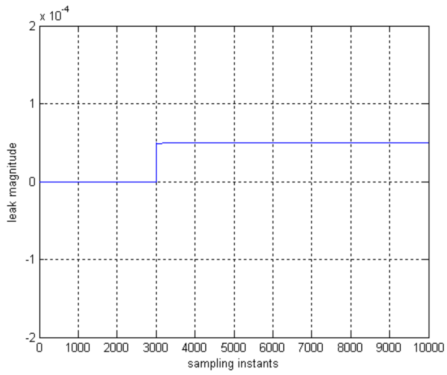

Fig. 5 – *Structured residual1 of the system*

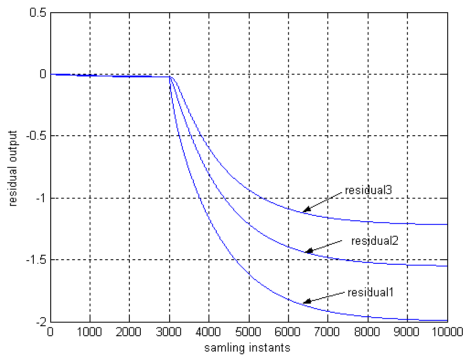

Fig. 6 – *Structured residual2 of the system*

Leak of magnitude 50 [ml/s] is given in tank1 at t = 3000[s].

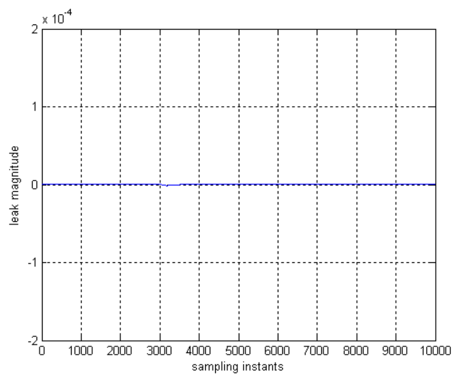

Fig. 7 – *Structured residual3 of the system.*

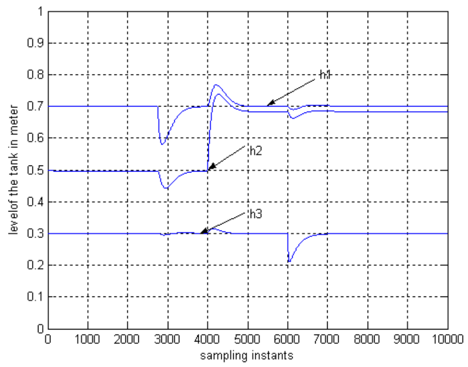

Fig. 8 – *Closed loop response of the system.*

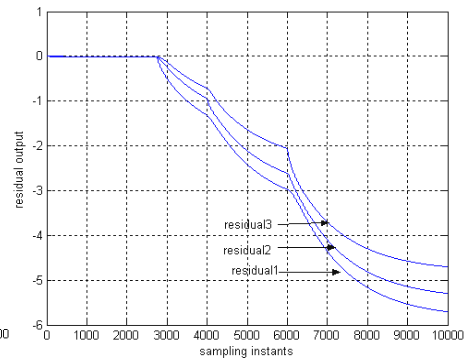

Fig. 9 – *Evolution of residuals.*

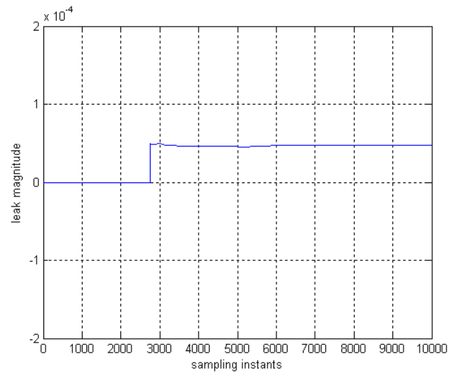

Fig. 10 – *Structured residual1 of the system.*

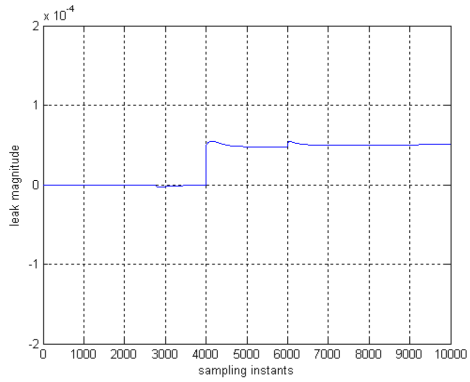

Fig. 11 – *Structured residual2 of the system.*

Leak of magnitude 50 [ml/s] is given in tank1 at t = 2750[s].
Leak of magnitude 50 [ml/s] is given in tank2 at t = 4000[s].
Leak of magnitude 100 [ml/s] is given in tank3 at t = 6000[s].

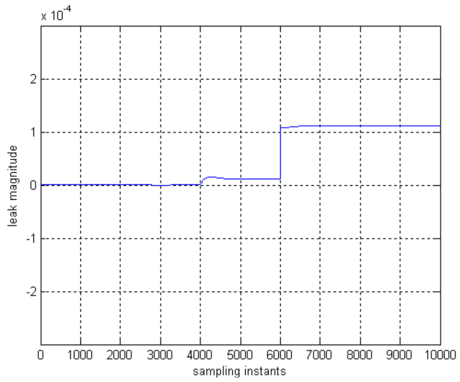

Fig. 12 – *Structured residual3 of the system.*

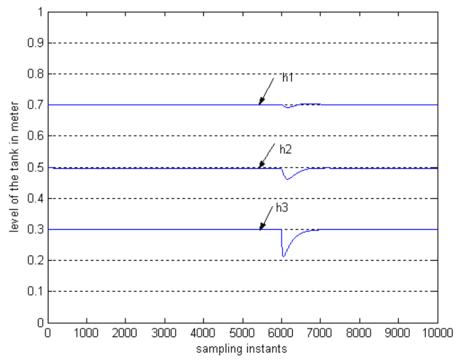

Fig. 13 – *Closed loop response of the system when a leak of magnitude* 50 [ml/s] *occurs in tank3 at* t = 6000[s].

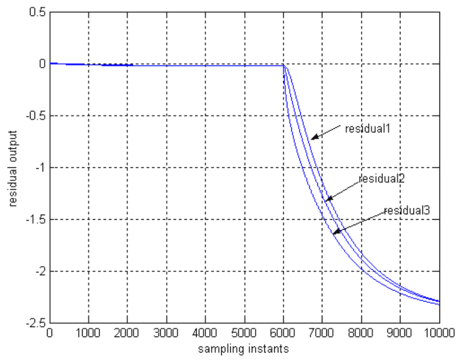

Fig. 14 – *Evolution of residuals when step change in leak of tank3 of magnitude* 50 [ml/s] *introduced at* t = 6000[s].

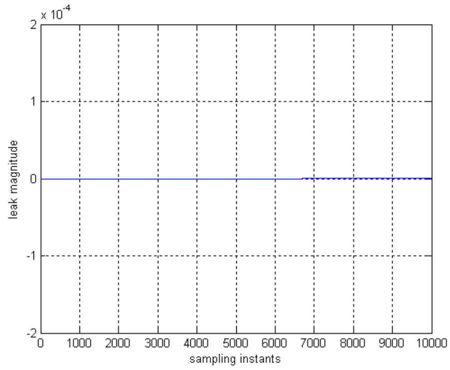

Fig. 15 – *Structured residual1 of the system*

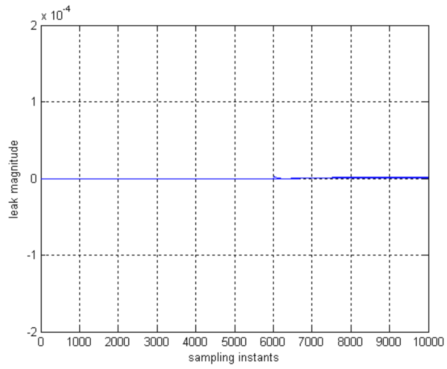

Fig. 16 – *Structured residual2 of the system*

Leak of magnitude 50 [ml/s] is given in tank3 at t = 6000[s].

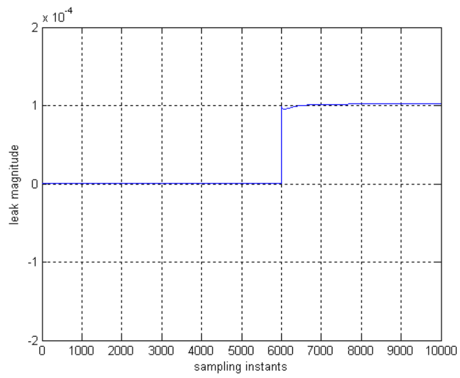

Fig.17 – *Structured residual3 of the system.*

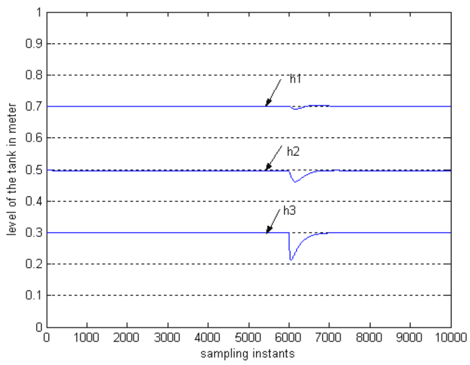

Fig. 18 – *Closed loop response of the system when a leak of magnitude* 100 [ml/s] *occurs in tank3 at* t = 6000[s].

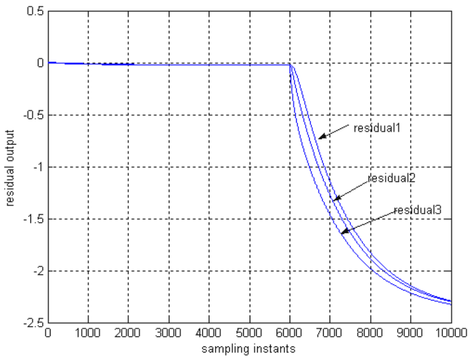

Fig. 19 – *Evolution of residuals when step change in leak of tank3 of magnitude* 100 [ml/s] *introduced at* t = 6000[s].

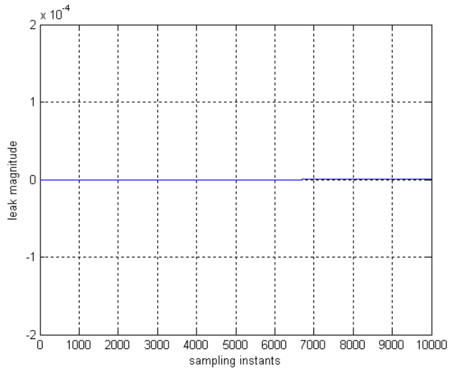

Fig. 20 – *Structured residual1 of the system.*

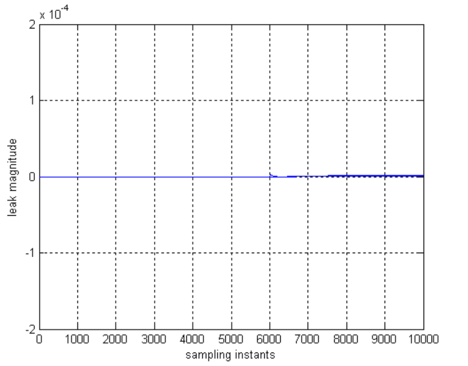

Fig. 21 – *Structured residual2 of the system.*

Leak of magnitude 100 [ml/s] is given in tank3 at t = 6000[s].

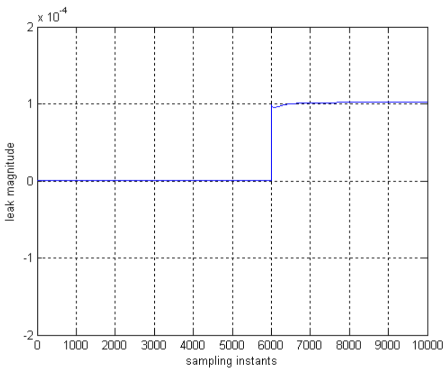

Fig.22 – *Structured residual3 of the system.*

6 Conclusions

The performance of the proposed scheme has been evaluated on a three-tank process for leak in the tanks. The proposed FDI scheme can provide fault information even when there is simultaneous change in more than one leak. It should be noted that the proposed method is independent of the controller design. From the structured residuals, the magnitude of leak and time of occurrence of leak are also found. And one can conclude that the estimated magnitude and time of occurrence of the leak variable (fault) are close to the true value. So from the proposed method one can identify the fault as soon as it occurs in the process. The proposed method is found to be robust to plant model mismatch.

7 References

[1] J. Gertler: Fault Detection of Dynamical Systems, Marcel Dekker, Inc. USA, 1998.

[2] J. Gertler: A Survey of Model Based Failure Detection and Isolation in Complex Plants, IEEE Control Systems Magazine, Vol. 8, No. 6, pp. 3-11, 1988.

[3] J. Gertler and M. Staroswiecki: Structured Fault Diagnosis in Mildly Nonlinear Systems: Parity Space and Input-Output Formulation, FAC 15th World Congress, Barcelona, Spain, July 2002.

[4] J. Gertler, M. Staroswiecki, M. Shen: Direct Design of Structured Residuals for Fault Diagnosis in Linear Systems, American Control Conference, Anchorage, Alaska, May 2002.

[5] J. Gertler: Residual Generation in Model Based Fault Diagnosis, Control-Theory and Advanced Technology, Vol. 9, pp. 259-285, March 1993.

[6] J. Wu, G. Biswas, S. Abdelwahed, E. Manders: A Hybrid Control System Design and Implementation for a Three Tank Testbed, Proceedings of the 2005 IEEE Conference on Control Applications, pp. 645–650, August 2005.

[7] E. Marlin: Process Control: Designing Processes and Control Systems for Dynamic Performance, McGraw-Hill International Editions 1995.

[8] G. Stephanopoulos: Chemical Process Control, An Introduction to Theory and Practice, Prentice-Hall India Publications 2000.

[9] J. Gertler: A Survey of Model Based Failure Detection and Isolation in Complex Plants, IEEE Control Systems Magazine, Vol. 8, No. 6, pp. 3-11, 1988.

[10] J. Gertler: Residual Generation in Model Based Fault Diagnosis, Control-Theory and Advanced Technology, Vol. 9, pp. 259-285, March 1993.

[11] J. Gertler: A Numerical-Structural Approach to Failure Detection and Isolation in Complex Plants, Proceedings of the IEEE Conference on Decision and Control, Athens, Greece, pp. 1576-1580, 1986.

[12] R. Isermann: Process Fault Detection Based on Modeling and Estimation Methods – a Survey, Automatica 20, pp. 387-404, 1984.

[13] B. Käoppen-Seliger, E. Alcorta-Garca, P.M. Frank: Fault Detection – Different Strategies for Modeling Applied to the Three Tank Benchmark – A Case Study, European Control Conference, Karlsruhe, Germany, 1999.

[14] E. Alcorta-Garca, P.M. Frank: Observer-Based Fault Detection via Control System Decomposition, SAMS, 24(51), pp. 177-184, 1996.

[15] P.M. Frank, X. Ding: Survey of Robust Residual Generation and Evaluation Methods in Observer-Based Fault Detection Systems, Journal of Process Control, 7(6), pp. 403-424, 1997.

[16] Q. Zhang, M. Basseville, A. Benveniste: Fault Detection and Isolation in Nonlinear Dynamic Systems: A Combined Input-Output and Local Approach, Automatica 34, pp. 1359-1373, 1988.

[17] D. Theilliol, H. Noura, J.C. Ponsart: Fault Diagnosis and Accommodation of a Three-Tank System Based on Analytical Redundancy, ISA Transactions 41, 2002.

[18] A. Akhenak, M. Chadli, D. Maquin, J. Ragot: State Estimation via Multiple Observer the Three Tank System, Proc. 5th IFAC SAFEPROCESS, Washington, 2003.

[19] J. Chen, R.J. Patton: Robust Model-Based Fault Diagnosis for Dynamic Systems, Kluwer Academic Publishers, ISBN 0-7923-8411-3, 1999.