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A Transformer Model Based on the Jiles–Atherton Theory of Ferromagnetic Hysteresis

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Abstract: This paper presents a transformer model that is useful for low-frequency applications. To describe the iron-core magnetic behavior, the Jiles Atherton hysteresis model is used, which is able to generate minor asymmetric loops and remanent flux. The obtained results are compared with those measured in the laboratory on a commercial resistance welding transformer.

Keywords: Jiles-Atherton, Transformer, *B*(*H*), Simulation.

1 Introduction

For a long time there has been a search for a general transformer model, i.e. a model capable of predicting the transformer's behavior over a wide frequency range and for all possible load situations. Such a model could be incorporated as a black box in a power system analysis package like EMTP or SPICE. The user of the program would then no longer have to worry about the validity of the model.

Different models can be used for different frequency ranges and different loading situations of the transformer. An overview of models is given in [1]. As long as the transformer is loaded and the frequency of interest is low, fairly simple models can be used: leakage reactance, copper losses, winding capacitances.

For non-loaded transformers the non-linear behavior of the transformer core has to be taken into account. As long as the magnetizing current is more or less sinusoidal, including hysteresis and saturation will lead to acceptable results. But for cases with non-sinusoidal currents or voltages no satisfactory model has been proposed yet. Examples are the inrush current in no-loaded transformers, ferro-resonant overvoltages, and the subject of this paper: thyristor control of welding transformers. What makes this especially arduous is the combination of non-linear and frequency dependent effects. Where the former calls for a time-domain model the latter requires a more frequency-domain oriented approach as

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e.g. used in transmission line modeling. The more non-sinusoidal the voltage or current, the more high frequencies are present and the more the frequency dependent effects have to be taken into account. Note that because of the non-linearity a sinusoidal excitation no longer guarantees a sinusoidal response. The frequency range of interest can thus only be determined after an initial study.

2 The Jiles-Atherton Model

Jiles and Atherton [2, 3] describe the non-linear core based on physical properties of the magnetic material, using the current physical theories of magnetic domains in ferromagnetic materials. The Jiles-Atherton model requires the following input parameters: magnetization saturation, thermal energy parameter, domain flexing constant, domain anisotropy constant, interdomain coupling parameter. These are not the parameters that transformer manufacturers or manufacturers of transformer steel can provide. In fact they cannot even be determined directly through measurements. The various core hysteresis parameters required in this model are theoretical and can be calculated from experimental measurements of the coercivity, remanence, saturation flux density, initial anhysteretic susceptibility, initial normal susceptibility, and the maximum differential susceptibility. This is an iterative trial-and-error process [4, 5]. A parameter that is ill-defined by the 50 Hz curve could have a big influence on e.g. the magnitude of the inrush current.

Other core parameters that are needed to model a transformer core using this method are the magnetic cross section of the core, the magnetic path length, and the core stacking factor for laminated cores.

3 Simulation of a Transformer's $B(H)$ Loop

The transformer model in this paper is based on the Jiles-Atherton (JA) phenomenological model of a ferromagnetic core. Some commercially available programs [6] use the JA model to simulate the dynamic behaviour of magnetic devices.

The JA model has been applied for simulation of a commercial resistance welding transformer. According to the manufacturer data and the measurements performed, the resistance welding transformer has the following rated data: primary voltage 380 V; secondary no-load voltage (1.41-4.63) V; conventional power 24 kVA; rated frequency 50 Hz; thyristor controlled switching; number of primary tap positions 9. The transformer is a single phase with a shell type core. The parameters of the JA model were estimated such that the measured loop $B(H)$ and the saturation characteristic are reasonably accurately produced using the V_{rms}/I_{rms} model.

An example of a simulated $B(H)$ loop for the resistance welding transformer core is shown in Fig. 1. In this simulation a sinusoidal current was used to excite the primary winding with the secondary open circuited.

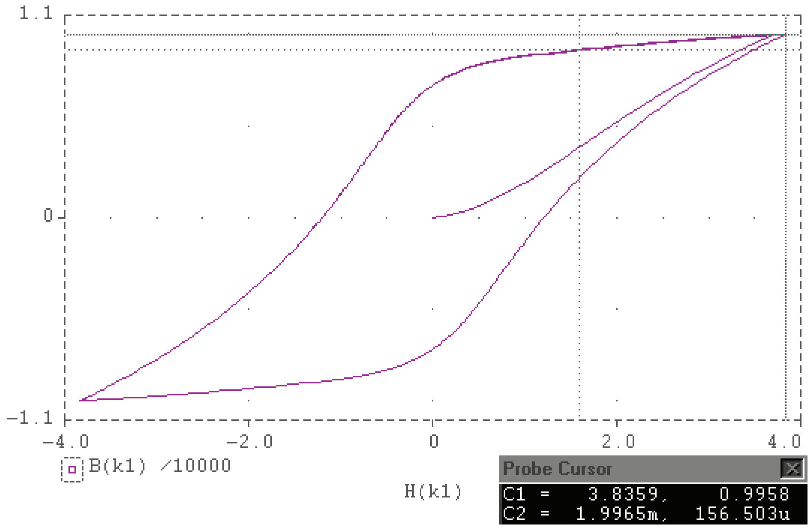

Fig. 1 – *Simulation of B(H).*

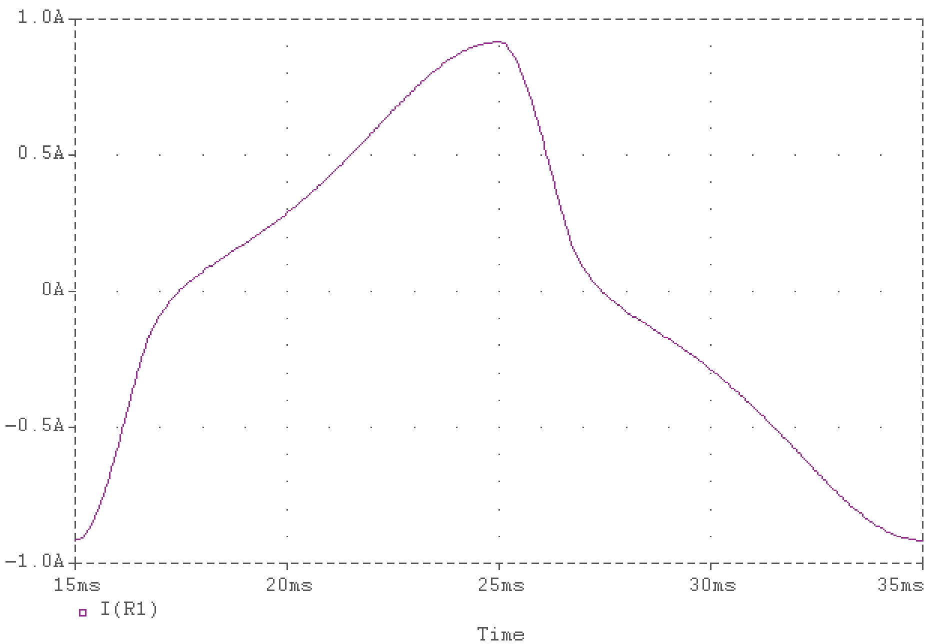

Fig. 2 – *Switching transients simulation* - $\beta = 0$.

For the simulation the following parameters were used: Primary turns 150; Secondary turns 1; Mean magnetic core area 118cm^2; Mean magnetic path length 45cm; Core pack factor 0.95; Effective air-gap length GAP = 0cm. The following theoretical parameters were determined: Magnetization saturation: 2.05×106A/m; Thermal energy parameter 250 [A/m]; Domain flexing constant 0.4; Domain anisotropy constant 320 [A/m]; Interdomain coupling parameter 2.8E-4. The core parameters were obtained directly from the transformer geometry apart from the theoretical JA parameters. The theoretical parameters were obtained in an iterative process by matching the simulated hysteresis loop to the one obtained experimentally. The simulated magnetizing current is shown in Fig. 2.

Simulated magnetic properties of the resistance welding transformer core are given in **Table 1** together with the measured core parameters for comparison.

Table 1

Measured and simulated magnetic properties of the transformer core.

	Measur.	Simulat.	Relat. error
Excitation current magnitude [A]	1,000	0,916	–8,40
Excitation current r.m.s. [A]	0,563	0,562	–0,18
Saturation induction [T]	0,967	0,996	3,00
Field at loop tip [A/m]	310	306	–1,29
Remanence [T]	0,68	0,71	4,41
Coercivity [A/m]	112	96	– 14,2
Iron losses [W]	100	100	0,00
Magnetizing resistance [Ω]	315	316	0,32
Magnetizing reactance [Ω]	597,0	597,7	0,12

The selected results show extremely good agreement between simulation and laboratory test data.

4 Comparison of Simulation and Test Results

4.1 Steady-state study

The resistance welding transformer model based on the JA hysteresis model was tested in steady-states, both for sinusoidal operation and for non-sinusoidal discontinuous operation due to primary side phase control. **Table 2** compares steady-state results at load conditions for sinusoidal operation of the transformer.

Table 2

Measured and simulated current at sinusoidal conditions.

Current	Measured	Simulated	Relative error %
I_1 [A]	54,2	56,0	3,32
I_2 [A]	8130	8347	2,67

Table 3 compares measurements and simulation for the case of primary side phase (thyristor) control of the tested transformer, for different firing angles. The comparison of the simulated and measured current for a firing angle of 3.48 ms is presented in Fig. 3.

The steady-state results achieved with the transformer model based on the Jiles–Atherton theory of ferromagnetic hysteresis show very good agreement with test results, for different load situations as well as different power supply conditions.

Table 3

Measured and simulated current for non-sinusoidal conditions.

Firing angle (ms)	Current	Measured	Simulated	Relative error %
3,48	I_1 [A]	23,29	23,73	1,9
	I_2 [A]	3711	3469	–6,5
4,28	I_1 [A]	20,01	19,98	0
	I_2 [A]	3134	2985	–4,75
5,17	I_1 [A]	15,75	15,74	0
	I_2 [A]	2470	2367	–4,17
5,98	I_1 [A]	12,79	12,09	–5,5
	I_2 [A]	1793	1774	–1,0

With these results the JA model has been confirmed to be very accurate for sinusoidal, non-sinusoidal, as well as discontinuous operation.

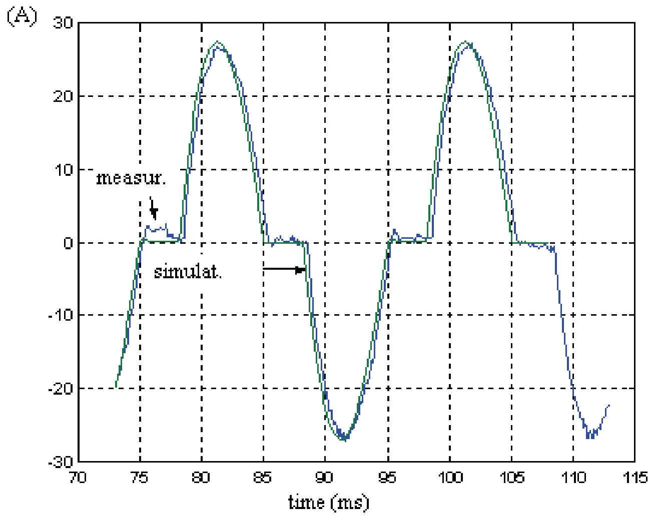

Fig. 3 – *Measured and simulated current in the case of phase control.*

4.2 Transient Study

Next, the resistance welding transformer model based on the JA theory of ferromagnetic hysteresis was tested in transient operation. Transients have been studied for two different cases, both driving the transformer core into saturation. First, an inrush transient study was performed. The transient performance of the transformer at no-load was modeled by defining zero transition of the winding voltage β=0. Sample results for the transient performance of the analyzed transformer are shown in Fig. 4. The corresponding *B*(*H*) loop is presented in Fig. 5. Similarly, Fig. 6 and Fig. 7 show simulation results for the switching angle of $\beta = \pi/4$.

From the worst-case transient switching results ($\beta = 0$) in Fig. 4 the exponentially decaying dc component of the primary current can be clearly observed. The current waveform displays a peak at the beginning. The simulated inrush current was compared with the corresponding measured inrush. However the simulation results showed insufficient increase of the inrush current. Where the measurements showed a current up to 90 times bigger then the steady-state current at no load, the simulation based on the JA transformer model showed an increase of 4 times only. When interpreting the experimental results one has to keep in mind that the measured value is approximative. The core may retain an unknown amount of remanent flux that built-up such a large inrush current, during the subsequent transformer switching.

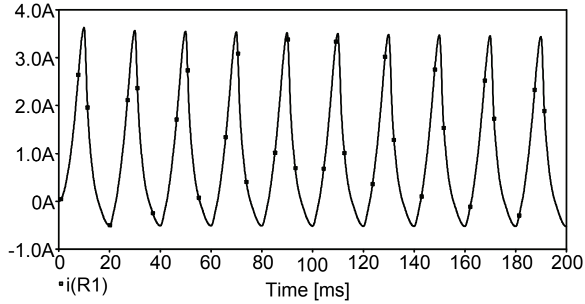

Fig. 4 – *Switching transients simulation* - $\beta = 0$.

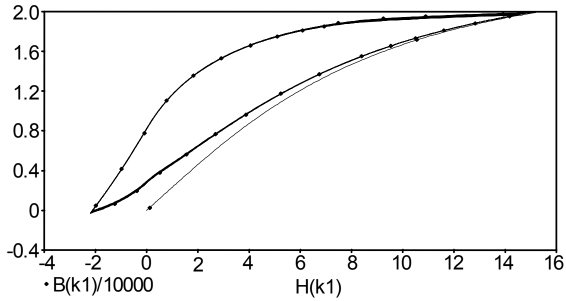

Fig. 5 – *Switching transients simulation of* $B(H)$ *loop* - $\beta = 0$.

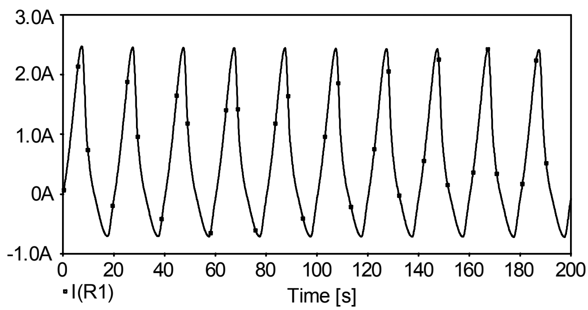

Fig. 6 – *Switching transients simulation* - $\beta = \pi / 4$.

Studying a different saturation case, by suppressing the triggering pulse of one of the thyristors of the phase control, it was found that the JA model overestimated the increase of the simulated primary current compared to the measured, by a factor of two. The simulated primary and secondary current shapes are presented in Fig. 8. The variation of the core flux due to DC magnetization is shown in Fig. 9.

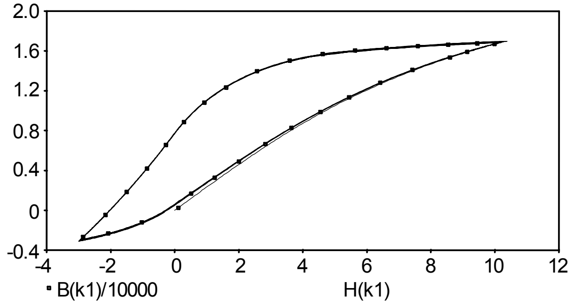

Fig. 7 – *Switching transients simulation of* $B(H)$ *loop* – $\beta = \pi/4$.

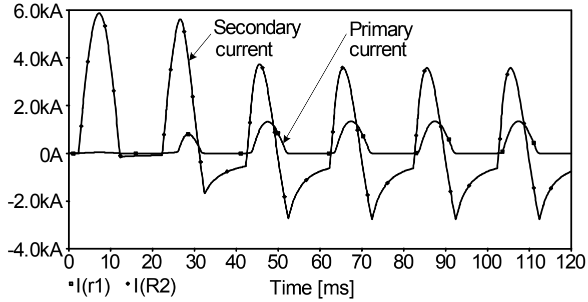

Fig. 8 – *Primary and secondary current – heavy saturation.*

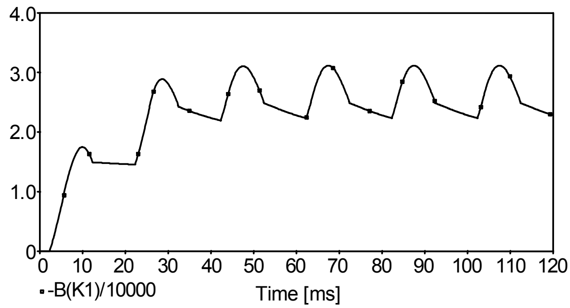

Fig. 9 – *B*(*t*) *due to* DC *magnetization*.

The waveforms for currents, flux and *B*(*H*) obtained by simulation are qualitatively accurate representations of the phenomenon measured on the test plant. The quantitative disagreement could be due to the fairly simple transformer model used (one leakage reactance plus one magnetizing reactance). Also, a JA parameter that is ill-defined by the 50 Hz curve could have a big influence on the magnitude of the currents.

These results indicate that the Jiles-Atherton model, despite its physical detail, remains a quasi-DC model, valid for slow variation only. The model can not follow the fast changes happening in the core when saturation effects start to play a role. The highly non-sinusoidal current in the case of saturation contains a significant amount of higher harmonics. The damping of these is not incorporated correctly.

5 Simulation Model Study

Commercial simulation packages that use the JA transformer model can be used for studying various engineering problems concerning transformers. Some suggestions on the practical application of the JA transformer model are briefly summarized bellow:

1) Power/energy transfer in the transformer in the case of sinusoidal operation and in the case of phase control: The PSPICE program has options to calculate accurately rms and avg of the time varying functions enabling determination of the powers and energy at any time instant and any place in the model. This can be particularly interesting for studying the increased distribution power losses in the case of phase control due to the current shape distortion.
2) Impact of the inserted magnetic materials over the equivalent cosφ of the equipment for resisting heating/welding: Inserting large sized components and/or magnetic materials in the secondary (welding) circuit have an impact on the secondary impedance and thus on the equivalent phase angle. The PSPICE simulation model enables prediction of the whole welding system behavior in the case of a sudden change of the equivalent phase angle.
3) Evaluation of the transformer magnetizing circuit parameters despite these are not explicitly shown in the simulation model: The results from **Table 1** show that despite the fact that the PSPICE model does not include the magnetizing circuit in explicit form, its parameters could be determined and evaluated. The practical application of magnetizing circuit parameters evaluation is seen to be interesting in the case of a transformer at no load, when the influence of the magnetizing circuit is dominant.
4) Harmonic analyses in the case of phase control: The program has options to calculate the harmonic components of the time varying functions and enables deepened harmonic analyses.

6 Conclusion

A transformer model based on the Jiles-Atherton theory of ferromagnetic hysteresis is valid for an iron-core transformer, regardless of its size, rating, working condition and power system topology as long as the transformer is not driven into heavy saturation. These results have been verified with laboratory test data obtained for a commercial resistance welding transformer.

Some suggestions on the practical application of the JA transformer model are included.

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