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# Forces in a 3D Magnetic Field of Conducting Current Contours

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**Abstract:** The present paper deals with 3D magnetic field analysis of conducting current contours. The magnetic field and forces were calculated analytically and by FEM applying the Comsol Multiphysics package. Forces were calculated by the Maxwell stress tensor and by volume force density. Numerical results for real and ideal contours with the same linear dimensions are discussed. Comparison between analytical and numerical data shows satisfactory agreement.

**Keywords:** Electrodynamic force, 3D magnetic field, Analytical solution, FEM computation.

## 1 Introduction

In industrial and physics applications conducting current contours have an arbitrary shape. Precise evaluation of the forces between them and the pressure on them is of important significance. The 3D magnetic field and forces acting on contours can be calculated applying FEM and the Comsol Multiphysics package. Forces are computed numerically and analytically considering contours as ideal hard bodies. Analytical results are based on determination of self and mutual inductances [1] and the magnetic energy. By the help of the virtual work principle the forces exerted on the contours are determined as numerical derivatives of the mutual magnetic energy of the system. Examples refer to the number of rectangular conducting current real and ideal contours.

## 2 System of Interacting Contours

An interacting system of contours is shown in Fig. 1. It consists of three coaxial rectangular contours with parallel sides denoted by *a* and *b*.

Dimensions are as follows: $a_1 = 10\,\text{cm}$ and $b_1 = 20\,\text{cm}$ for the first, $a_2 = 20\,\text{cm}$ and $b_2 = 40\,\text{cm}$ for the second, $a_3 = 5\,\text{cm}$ and $b_3 = 10\,\text{cm}$ for the third contour. They are located normally to the axis of symmetry $x$. Along contours 1 and 2 flowing currents have the same directions but in 3 the current flows opposite to them. The current values are: $i_1 = 10\,\text{A}$, $i_2 = 15\,\text{A}$ and $i_3 = 20\,\text{A}$

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respectively. The positions of 1 and 2 are fixed at a distance $x_{12} = 10\,\text{cm}$. Only the third contour is mobile.

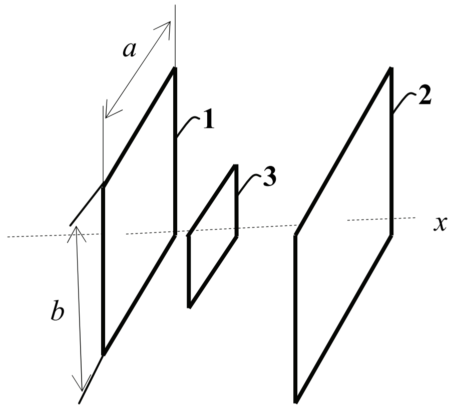


**Fig. 1** – *Interacting contours.*

The main purposes of this work are:

- Analytical and power FEM package 3D force calculation. Comparison between analytical and numerical results. Investigation of the possibilities of using a similar system as a testing example of different packages in 3D computation.
- Determination of the system equilibrium state.
- Comparison between the numerical results on the basis of real and ideal contours.

## 3 Analytical Solution

Reference [1] gives a very large number of formulas for inductance calculation of different contour configurations and mutual positions. Applying them, mutual inductances of contour pairs (fixed - mobile) can be calculated.

The mutual inductance of two coaxial rectangular contours with parallel sides is calculated applying the following term

$$M = \frac{\mu_0}{2\pi}[(a_1 + a_2)\ln p - (a_2 - a_1)\ln q +$$
$$+(b_1 + b_2)\ln p_1 - (b_2 - b_1)\ln q_1 - 4(v - w + v' - w')],$$

where:

$$p = \frac{a_1 + a_2 + 2v}{a_1 + a_2 + 2w} \cdot \frac{t'}{t}, \quad q = \frac{a_2 - a_1 + 2w'}{a_2 - a_1 + 2v'} \cdot \frac{t'}{t},$$

$$p_1 = \frac{b_1 + b_2 + 2v'}{b_1 + b_2 + 2w} \cdot \frac{u'}{u}, \quad q_1 = \frac{b_2 - b_1 + 2w'}{b_2 - b_1 + 2v} \cdot \frac{u'}{u}.$$

The quantities $p$, $q$, $p_1$, $q_1$ and also $u, v, w, t, u', v', w', t'$ are geometric parameters, defining distances between contour points. Applying the above expression, the mutual inductances $M_{13}$ and $M_{23}$ were calculated for different distances $x_{13}$ and $x_{23}$ between the contours. Some of the obtained values are:

$$M_{13} = 0.335 \cdot 10^{-7}\ \mathrm{H} \quad \text{and} \quad M_{23} = 0.152 \cdot 10^{-7}\ \mathrm{H},$$

for $x_{13} = 3\,\mathrm{cm}$ and $x_{23} = 7\,\mathrm{cm}$;

$$M_{13} = 0.218 \cdot 10^{-7}\ \mathrm{H} \quad \text{and} \quad M_{23} = 0.182 \cdot 10^{-7}\ \mathrm{H},$$

for $x_{13} = 5\,\mathrm{cm}$ and $x_{23} = 5\,\mathrm{cm}$;

Contours are considered solid bodies. Applying the principle of virtual work the forces between a mobile and corresponding fixed contour are determined by the terms

$$f_{13} = i_1 i_3 \frac{\partial M_{13}}{\partial x_{13}} \quad \text{and} \quad f_{23} = i_2 i_3 \frac{\partial M_{23}}{\partial x_{23}}. \tag{1}$$

For avoiding additional errors in numerical determination of the derivatives and to improve accuracy of the results two approximations describing mutual inductances were introduced. Approximations are presented by the expressions of the following mode:

$$M_{13} : y^{-1} = a + bx + cx^2 + dx^3 + ex^4 + fx^5$$

$$M_{23} : y^{0.5} = \frac{a + cx + ex^2}{1 + bx + dx^2}$$

for approximation 1 and

$$M_{23} : y^{0.5} = a + bx^2$$

$$M_{23} : y^{0.5} = \frac{a + cx + ex^2}{1 + bx + dx^2}$$

for approximation 2.

Instead of using finite increments ( $f_{13} = i_1 i_3 \frac{\Delta M_{13}}{\Delta x_{13}}$, $f_{23} = i_2 i_3 \frac{\Delta M_{23}}{\Delta x_{23}}$ ) derivatives are now determined analytically. In this manner a serious precondition for errors is prevented.

Linearity of the investigated region enables the resulting force determination as a sum

$$F = f_{13} + f_{23}\,. \qquad (2)$$

Practically the two forces are subtracted because one of them is negative. For the above distances the calculation results applying finite increments are as follows:

$$f_{13} = -7.83 \cdot 10^{-5}\ \mathrm{N}\,,\ f_{23} = -4.34 \cdot 10^{-5}\ \mathrm{N} \quad \text{and}$$

$$F_{\mathrm{f.i.}} = -3.49 \cdot 10^{-5}\ \mathrm{N}\ (x_{13} = 3\,\mathrm{cm}\ \text{and}\ x_{23} = 7\,\mathrm{cm});$$

$$f_{13} = -9.36 \cdot 10^{-5}\ \mathrm{N}\,,\ f_{23} = 4.44 \cdot 10^{-5}\ \mathrm{N} \quad \text{and}$$

$$F_{\mathrm{f.i.}} = -4.92 \cdot 10^{-5}\ \mathrm{N}\ (x_{13} = 5\,\mathrm{cm}\ \text{and}\, x_{23} = 7\,\mathrm{cm}).$$

For 11 different positions of the mobile contour the obtained data are shown in **Table 1**. The optimization procedure with respect to the variable $x$ of the obtained unidimensional dependence gives the result $F = 0\,\mathrm{N}$ for $x_{13} = 0.28\,\mathrm{cm}$. Only then the forces exerted to the mobile contour 3 are equal: $f_{13} = -3.5 \cdot 10^{-5}\ \mathrm{N}$ and $f_{23} = 3.5 \cdot 10^{-5}\ \mathrm{N}$.

## 3 Numerical Solution

Approximately the same results were numerically obtained using FEM. The magnetic field and forces were calculated applying the Comsol Multiphysics package [2]. Contours are surrounded by a large buffer zone as the boundary condition magnetic field does not penetrate the top and side walls.

Investigations were made on real and ideal contours. “Real” contours have the same middle geometric longitudinal lines and $1\,\mathrm{cm}^2$ square cross sections. Ideal contours have a cross section of $1\,\mathrm{mm}^2$. The linear dimension of this section is negligible with respect to every one of the longitudinal linear dimensions of contours. For the same 11 positions of the mobile contour 3 the resulting force is calculated on the basis of the Maxwell stress tensor $F_{MST}$ and the volume force density $F_{v.f.d.}$. Contours are closely included in the air block. The magnetic energy of the whole system was calculated applying the subdomain integration of the magnetic energy density over the surface of the embracing block. Calculated forces and the magnetic energy of the system and the relation between the quantity of the magnetic energy and the equilibrium state of the system were checked.

### 3.1 Short review of field characteristics

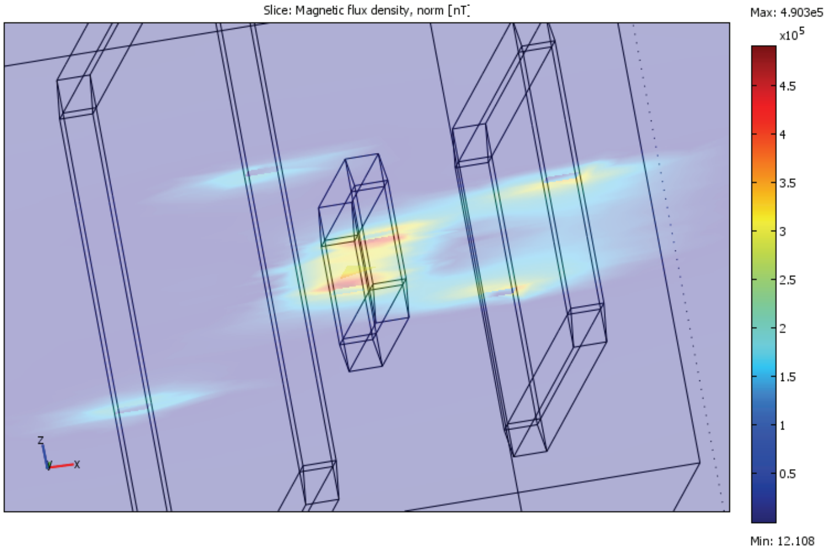


**Fig. 2** – *Magnetic flux density norm at a plane* $z = 0$, *for real contours at a distance* $x_{13} = x_{23} = 5\,\text{cm}$.

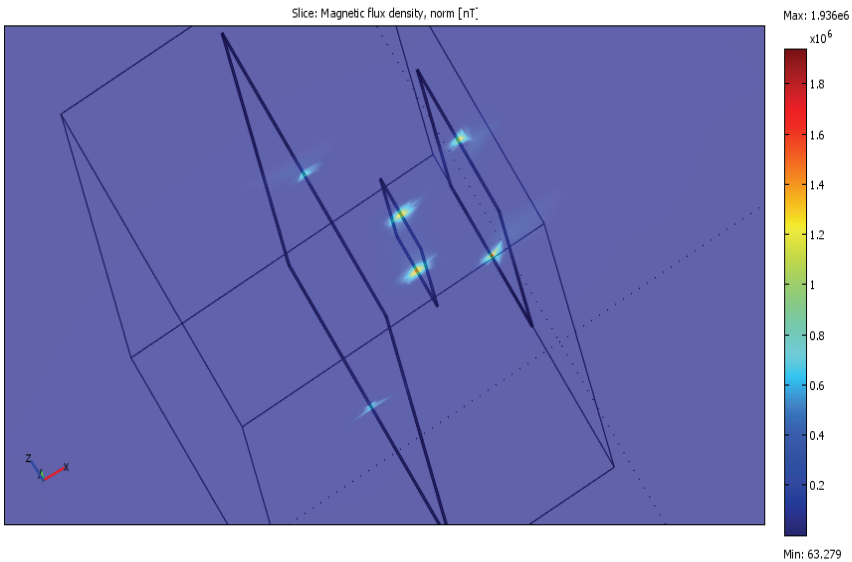


**Fig. 3** – *Magnetic flux density norm at a plane* $z = 0$, *for ideal contours at a distance* $x_{13} = x_{23} = 5\,\text{cm}$.

Field information is obtained from the values and deviations of the main characteristics: magnetic flux density norm, Maxwell surface stress tensor and the volume force density. The magnetic flux density norm is small (Fig. 2 - for real and Fig. 3 - for ideal contours) because of the small contour dimensions, low currents and nonmagnetic medium around.

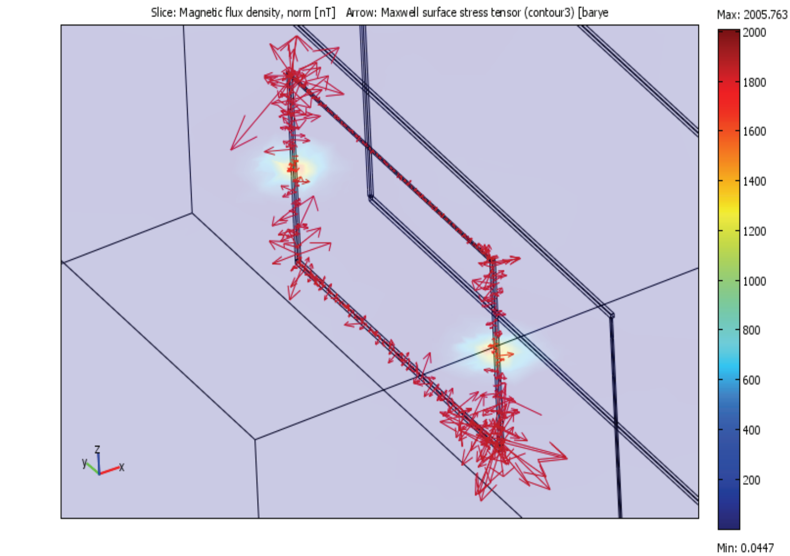


**Fig. 4** – *Maxwell surface stress tensor, contour* 3.

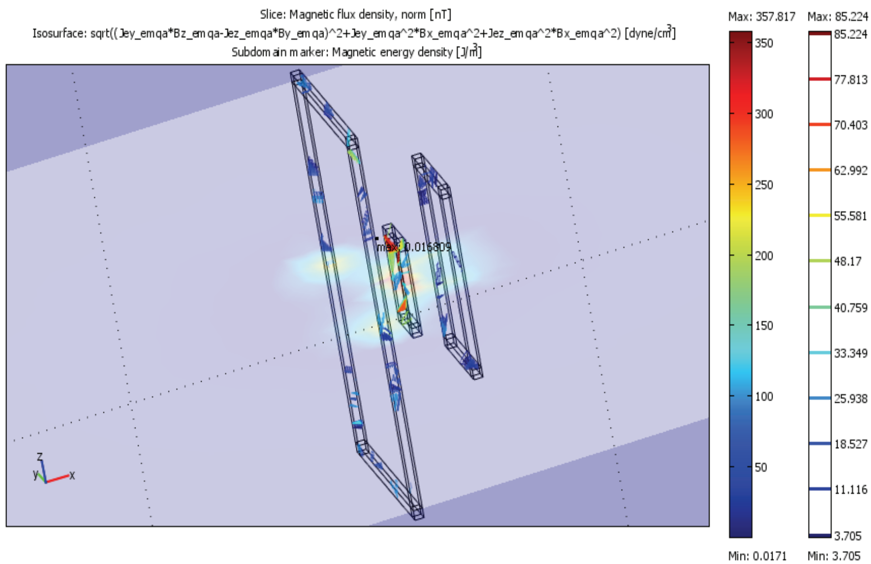


**Fig. 5** – *Isosurfaces of the volume force density*.

The Maxwell surface stress tensor (Fig. 4) and the volume force density (Fig. 5) are strongly non-uniformly distributed along the conductors.

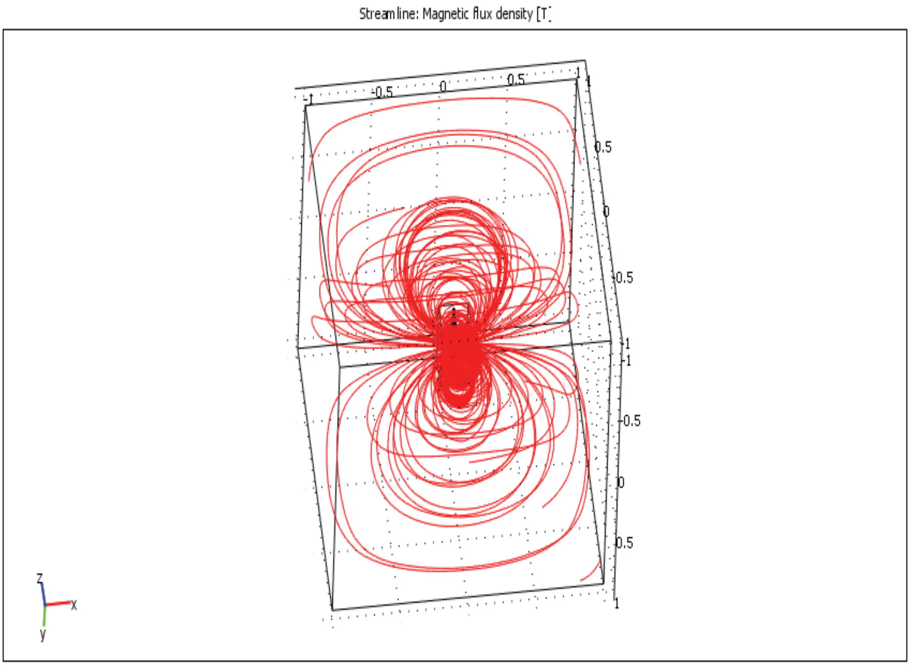


**Fig. 6** – *Streamlines: Magnetic flux density*.

The large rate of these uneven distributions confirms how important calculation is, not only for the global but the distributed local forces as well.

The streamline distribution is given in Fig. 6.

### 3.2 Force calculation

Numerical results obtained for the forces, calculated in the presence of real contours and for ideal contours as well are presented in **Table 1**. The included symbols denote as follows: *d*-distance between the moving and the fixed first contour; $F_{\mathrm{MST}}$ and $F_{\mathrm{v.f.d.}}$ are the resulting forces acting on the moving contour for cases when contours are real or ideal; $F_{\mathrm{appr.1}}$ and $F_{\mathrm{appr.2}}$ are analytically calculated forces where the first or second approximation for mutual inductances was applied, respectively.

**Table 2** shows absolute deviations between calculated $F_{\mathrm{MST}}$ or $F_{\mathrm{v.f.d.}}$ and analytically determined forces depending on the approximation used 1 ($\varepsilon_f$) and 2 ($\varepsilon_s$). Relative errors were not evaluated because as the force values are close to zero. Real estimation results will be determined in a similar way.

The obtained results are properly discussed.

The dependencies are presented graphically. In Fig. 7 forces $F_{\text{MST}}$, $F_{\text{v.f.d.}}$ and $F_{\text{appr.1}}$, $F_{\text{appr.2}}$ are plotted from point to point by smooth lines in the $\times 10^{-5}$ N dimension. In Fig. 8 the absolute force deviations are given.

Because of the current directions the forces $f_{13}$ and $f_{23}$ acting at the mobile contour 3 are opposite and a distance exists for which the resulting force is null. Then the system is in equilibrium state.

**Table 1**

*Electrodynamic forces on the movable contour.*

| d [cm] | Real contours | | Ideal contours | Analytical solution | |
|---|---|---|---|---|---|
| | $F_{\text{MST}}$ [$\times 10^{-5}$ N] | $F_{\text{v.f.d.}}$ [$\times 10^{-5}$ N] | $F_{\text{MST}}$ [$\times 10^{-5}$ N] | $F_{\text{appr.1}}$ [$\times 10^{-5}$ N] | $F_{\text{appr.2}}$ [$\times 10^{-5}$ N] |
| 0 | –0.39 | 2.92 | 1.18 | 3.47 | 3.47 |
| 1 | –1.74 | –3.47 | –2.82 | –6.67 | 0.98 |
| 2 | –2.19 | –3.56 | –2.14 | –10.30 | –1.40 |
| 3 | –0.95 | –5.05 | –3.15 | –9.44 | –3.58 |
| 4 | –2.47 | –6.20 | –5.83 | –7.17 | –5.47 |
| 5 | –4.26 | –8.13 | –5.88 | –4.95 | –6.98 |
| 6 | –7.05 | –10.80 | –7.60 | –3.29 | –8.00 |
| 7 | –5.38 | –13.80 | –7.93 | –2.30 | –8.60 |
| 8 | –7.99 | –15.70 | –9.47 | –1.97 | –8.55 |
| 9 | –2.49 | –12.80 | –26.00 | –2.23 | –7.77 |
| 10 | –1.69 | –11.90 | –18.22 | –2.96 | –6.00 |

**Table 2**

*Absolute deviations.*

| d [cm] | Real contours | | | | Ideal contours | |
|---|---|---|---|---|---|---|
| | MST | | v.f.d. | | MST | |
| | $\varepsilon_f$ | $\varepsilon_s$ | $\varepsilon_f$ | $\varepsilon_s$ | $\varepsilon_f$ | $\varepsilon_s$ |
| 0 | –3.86 | –3.86 | –0.55 | –0.55 | 2.29 | –2.29 |
| 1 | 4.93 | –2.72 | 3.20 | –4.45 | 3.85 | –3.80 |
| 2 | 8.11 | –0.79 | 6.74 | –2.16 | 8.16 | –0.74 |
| 3 | 8.49 | 2.63 | 4.39 | –1.47 | 6.29 | 0.43 |
| 4 | 4.70 | 3.00 | 0.97 | –0.73 | 1.34 | 0.36 |
| 5 | 0.69 | 2.72 | –3.18 | –1.15 | –0.93 | 1.10 |
| 6 | –3.76 | 0.95 | –7.51 | 2.80 | –4.31 | –0.40 |
| 7 | –3.08 | 3.22 | –11.5 | –5.20 | –5.63 | 0.67 |
| 8 | –6.02 | 0.56 | –13.7 | –7.15 | –7.50 | –0.92 |
| 9 | –0.26 | 5.28 | –10.6 | –5.03 | –23.8 | –18.2 |
| 10 | 1.27 | 4.31 | –8.94 | –5.90 | –15.3 | –12.2 |

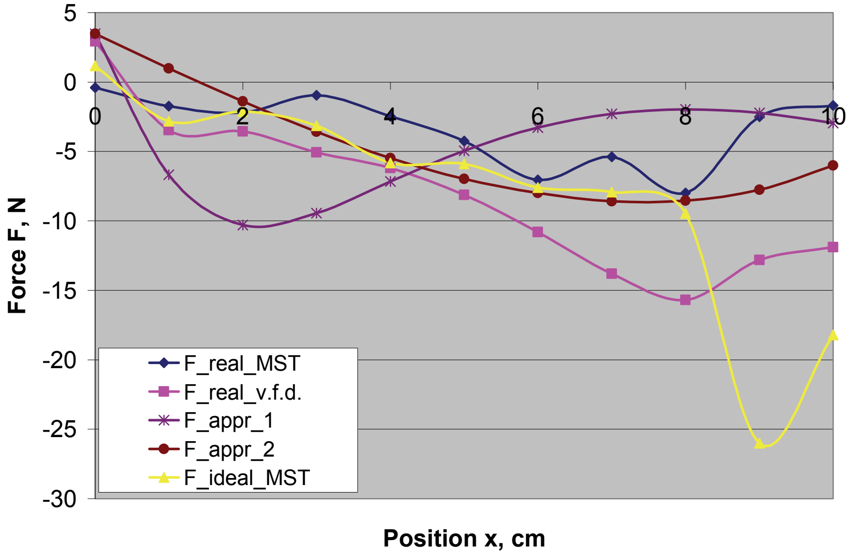


**Fig. 7** – *Dependencies for force F*(*x*) [N].

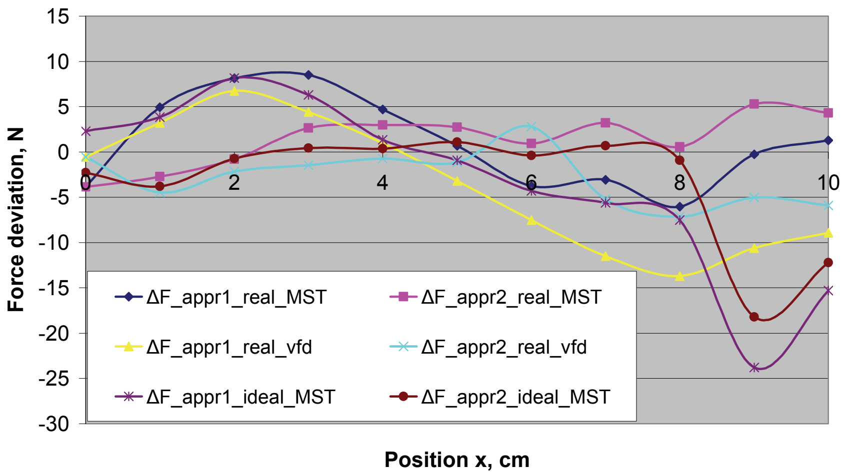


**Fig. 8** – *Deviations in absolute units between analytical and computational results.*

The values of the resulting force acting at the moving contour along the *x*-axis analytically and numerically calculated by the Comsol Multiphysics package are compared. The deviations in absolute units are presented graphically in Fig. 8.

Comparison between calculated and analytically determined field values at some arbitrary points in the surrounding area as well as the mutual inductance values between the contours show satisfactory agreement.

## 4 Conclusion

3D magnetic field analysis of conducting current contours was performed. The magnetic field and forces were calculated analytically and by FEM applying the Comsol Multiphysics package.

Analytical results were based on the virtual work principle.

The experience accumulated by working on this paper could be summarized as follows. 3D field calculations by the Comsol Multiphysics package require a very large physical memory. The use of a PC with 512 MB RAM enables calculation of the region including approximately 38000 elements applying only an extra coarse mesh.

Primary calculations using a system of real contours with the corners at right angles give a very large divergence (of $10^5$-$10^6$ times) between the values of $F_{\mathrm{MST}}$ and $F_{\mathrm{v.f.d.}}$. By changing the corners to the shape allowing application of the solenoidal external current density field the results closely converge.

Simultaneous calculations of the resulting forces on the moving contour and the magnetic energy of the system show the presence of a dependence. But its evaluation for practical applications needs additional precision.

Numerical results obtained based on the Maxwell stress tensor and the volume force density, are not only of the same rate but they are very close.

The force results for “real” contours (ratio 10 between the linear cross section and the minimal longitudinal dimension) and “ideal” contours (the value of the same ratio equal to 100) are practically uniformly useful. The results applying ideal contours are very close to the ones obtained by the first approximation for the first 80% of the distance between the fixed contours.

Investigations show that a similar system could be used as a testing example for different packages in 3D computation.

The optimization procedure with respect to the variable $x$ of the obtained unidimensional dependence $F = F(x)$, gives the result $F = 0\,\mathrm{N}$ at $x_{13} = 0.285\,\mathrm{cm}$ as the system equilibrium state.

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## 5 References

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