Improved Cuckoo Search Optimization for Production Inventory Control Systems

Huthaifa Al-Khazraji\textsuperscript{1}, William Guo\textsuperscript{2}, Amjad Jaleel Humaidi\textsuperscript{1}

Abstract: The efficient use of the production-inventory control system is of great importance in the industry. In this paper, an investigation study of the impact of an optimal Integral minus Proportional Derivative (I-PD) controller on the dynamic behavior of the production-inventory system is given. The differential equations of the system are first formulated. Then, the I-PD controller is proposed to enhance the responsiveness of the inventory performance. Two swarm optimization techniques, Cuckoo Search Optimization (CSO) and an Improved CSO (ICSO), are employed to tune the adjustable design parameters of the I-PD controller. In order to represent a realistic environment, two simulations are conducted. The first one is when the system is subject to a unit step change in the demand and second is when the demand is randomly. In a comparative performance study for both tuning schemes, the Integral of Time Absolute Error (ITAE) and the Integral of Square Error (ISE) indices were used as evaluation measures. The simulation outcomes using MATLAB show the superiority of the ICSO to tune the I-PD controller in terms of reducing the ITAE and ISE indices in comparison with the result obtained from traditional CSO. The result of this improvement indicates that the I-PD controller tuned by ICSO looks likely to better in terms of improving the performance of the system, particularly, by significantly reducing the inventory cost.

Keywords: Production-Inventory System, Control Theory, I-PD Controller, Swarm Optimization Technique, Cuckoo Search Optimization.

1 Introduction

The increasingly of the complicity of the business process has put a lot of pressure on the decision-maker to understand the dynamics of the supply chain. In this context, the purpose of production-inventory control systems is to ensure an efficient movement of information and goods between manufacturing and retailer within the business process [1]. In addition, increased competition in the business environment has encouraged production-inventory managers to seek for

\textsuperscript{1}Control and Systems Engineering Department, University of Technology- Iraq, Baghdad, Iraq; E-mails: 60141@uotechnology.edu.iq; amjad.j.humaidi@uotechnology.edu.iq
\textsuperscript{2}School of Engineering and Technology, Central Queensland University, Rockhampton, Queensland, Australia; E-mail: w.guo@cqu.edu.au
areas to improve firm profitability [2]. One way to reduce costs and to increase profitability is to reduce inventory levels as much as possible [3]. Due to uncertainties in the demand and the manufacturing environment, it is usually difficult for manufacturing companies to control production-inventory systems and achieve sustained cost-effective strategies.

Production-inventory problems have been successfully studied by control engineering practices over decades as an alternative way to model and control the performance of the system [4, 5]. It was found that these models adequately represent the industrial behavior and can be directly used by the managers to determine the required order rates in the face of the fluctuated market demand. Variety of studies and approaches of control theory that are applied to production-inventory problems could be found in [6–9].

In the context of controlling the production-inventory system, Towill [10] investigated the ability of the proportional controller to improve the performance of the production-inventory system. Later, Jonn et al. [11] and AL-Khazraji et al. [12] extended Towill's work by adding other feedback information to the system. Towill et al. [13] showed that the Proportional plus Integral (PI) controller could eliminate the inventory deficit in the production-inventory system. Tosetti et al. [14] and White and Censlive [15] implemented the Proportional-Integral-Derivative (PID) controller. Unlike the previous studies, the purpose of this study is to design an optimal Integral minus Proportional Derivative (I-PD) controller, one of the modified versions of the PID controller, for a production-inventory system. I-PD controller has been used in many control problems including level control system [16], ball and plate control system [17], speed control of a DC motor [18], control of a magnetic levitation system [19] and control the heating of the injection molding process [20]. In the same direction, the aim of this paper is to examine the ability of the I-PD controller to control the production-inventory system.

In order to design the I-PD controller, the differential equations of the production-inventory system are formulated to characterize the dynamics of the system. The objective of the I-PD controller is to ensure that the inventory level follows the desired level. Moreover, to improve the performance of the I-PD controller in terms of finding the best value of the adjustable parameters, researchers have utilized various optimization techniques such as bacterial foraging algorithm [21], particle swarm optimization [22], jaya algorithm [19] and firefly algorithm [20]. Each brought some improvements in the performance of the system to some extent but searching for better performance has been an ongoing endeavor. In this study, two swarm optimization techniques, Cuckoo Search Optimization (CSO) and an Improved CSO (ICSO) are used to fine tune the I-PD controller. Our simulation results show that I-PD controller tuned by ICSO reduced the deviation between the actual inventory level and the desired
inventory level to a lower level compared with that resulted from the I-PD controller tuned by CSO.

This paper reposts the design and mathematical presentation of the production-inventory system and the controller in Sections 2 and 3. The optimization mechanism to fine-tune the I-PD controller by both CSO and ICSO is presented in Section 4, the simulation results and discussions given in Section 5. Section 6 summaries the conclusions and the future work.

2 Mathematical Model

A production-inventory system is an essential unit in the supply chain that combines forecasting, orders decision, production process and the inventory as illustrated in Fig. 1 [5, 23]. The model was well documented by [10].

Fig. 1 - Production-inventory system.

The block diagram using Laplace Transformation of a single-stage single-product production-inventory system that is considered in this research is shown in Fig. 2. It is assumed that the demand rate for the product is \( D \). The state variable \( x_1 \) measures the level of the inventory. The state variable \( x_2 \) refers to the production rate. The process of production is modeled as a first-order lag with a time constant of \( T_p \). The time constant of the production process refers to the time required between receiving an order and delivering the item as a finished product [12, 11, 10]. The order rate \( u \) is the control input to the system. The decision maker of the manufacturing system needs to know the state variable \( x_3 \) which measures the average of the demand. The demand is forecasted by using a first-order lag with a time constant of \( T_a \) [24]. The time constant in the forecasting mechanism describes the average age of the data [25]. As a result, the level of inventory \( x_1 \), the production rate \( x_2 \), and the average demand rate \( x_3 \) change over time according to the following equations [26]:

\[
\dot{x}_1 = x_2 - D, \tag{1}
\]
\[
\dot{x}_2 = \frac{1}{T_p}(u - x_2), 
\]
\[
\dot{x}_3 = \frac{1}{T_a}(D - x_3).
\]

Fig. 2 – Block diagram of production-inventory system.

3 Controller Design

This section presents the design of the I-PD controller to improve the dynamics performance of the production-inventory system. The control strategy is designed based on forward and feedback information. The first objective of the controller is to ensure that the inventory level \( x_1 \) follows the desired level \( xd_1 \). The second objective of the controller is to ensure that the production rate \( x_2 \) follows the desired production rate \( xd_2 \). Therefore, the error \( e \) is defined as:

\[
e = e_1 + e_2,
\]

where \( e_1 \) is the error between the desired inventory level \( xd_1 \) and the actual inventory level \( x_1 \) and \( e_2 \) is the error between the desired production rate \( xd_2 \) and the actual production rate \( x_2 \). Therefore, (4) rewrites as follows:

\[
e = (xd_1 - x_1) + (xd_2 - x_2).
\]

In this work, the desired inventory level is set to zero in order to achieve the strategy of Just-in-Time (JIT). Besides, the desired production rate is designed to be equal to the average demand rate \( x_3 \). Substitute this in (5) gives:

\[
e = -x_1 - x_2 + x_3.
\]

The classical Proportional-Integral-Derivative (PID) controller provides effective solutions to a wide range of control engineering problems. It is a sum of three terms of the proportional (P) term, integral (I) term and derivative (D) [27]. The control signal \( u \) in the conventional PID controller is given by [28]:

\[
u = k_p e + k_i \int e \, dt + k_d \frac{de}{dt},
\]
where \( k_p, k_i \) and \( k_d \) are proportional gain, integral gain and derivative gain respectively. The conventional PID often suffers from what is called proportional and derivative kicks [21]. To overcome this drawback, a modified PID controller schematic known as I-PD is adopted in this study. Unlike the conventional PID, the proportional term in the I-PD changes the control signal proportionally to the output of the process. The integral term in the I-PD changes the control signal proportionally to the integration of the error which is the same as the conventional PID. However, the derivative term in the I-PD controller changes the control signal proportionally to the derivative of the output of the process. As a result, the control signal \( u \) in the I-PD controller is given by [29]:

\[
u = k_i \int e \, dt - k_p y + k_d \frac{d y}{d t},
\]

(8)

where \( y \) is the output of the process (i.e. inventory level). The block diagram of the I-PD controller is shown in Fig. 3.

![Fig. 3 – Block diagram of the I-PD controller.](image)

4 Swarm Optimization

This section presents the swarm optimization techniques that are used in this work to tune the adjustable parameters of the I-PD controller. Swarm optimization is one of the recent research topics in the field of artificial intelligence. Cuckoo Search Optimization (CSO) is a swarm optimization introduced by Yang and Deb in 2009 [30]. CSO aims to explore the search space of an optimization problem based on the breeding behavior of cuckoo birds [30]. The algorithm starts by initializing the population \((N)\) randomly within the lower and upper boundaries of the search space as given in (9) [31]:

\[
p_i = p_{\text{min}} + \text{Rand}(p_{\text{max}} - p_{\text{min}}),
\]

(9)

where:
Next, the CSO generates new solution based on Levy flights as given in (10):

$$p_{i}^{t+1} = p_{i}^{t} + \alpha \oplus L(\lambda),$$

(10)

where:
- $t$ – Index of the iteration
- $p_{i}^{t+1}$ – New solution
- $p_{i}^{t}$ – Current solution
- $\alpha$ – Step size
- $L(\lambda)$ – Levy distribution, $1 < \lambda < 3$.

In the CSO algorithm, the best solution is kept for the next generation and a fraction $\beta$ of the worse solution in the population is replaced by a new solution as given in (9).

In this work, an improved version of the CSO is proposed. In the context of swarm optimization algorithms, the concepts of exploration research and exploitation research represent distinct approaches that the algorithm employs to navigate a given problem space. Exploration research refers to the strategies of exploring new solution regions and exploitation research revolves around the known promising solutions. In the proposed improved version of the CSO, the adjustment position of each agent is influenced by the optimal positions identified by the algorithm. This adjustment mechanism is enabling the new algorithm to be more effective exploration and convergence toward optimal solutions. Therefore, the mechanism adjustment of the position of the individual agent in the CSO as given in (10) is replaced by (11) in the improved version of the CSO. The pseudo-code of the proposed CSO and ICSO is illustrated in Fig. 4.

$$p_{i}^{t+1} = p_{i}^{t} + \alpha \oplus L(\lambda)(p_{g} - p_{i}^{t}),$$

(11)

where $p_{g}$ is the best solution.

The convergence of the proposed algorithm is improved by using (11) in terms of adjusting the movement of the individual agent towards the best solution instead of using the random walk that is used by the basic version of the CSO [30].
Improved Cuckoo Search Optimization for Production Inventory Control Systems

1. **Input**
   - Objective function, Population size ($N$), Step Size ($α$), Probability ($β$), Number of Iteration ($T_{\text{max}}$)

2. **Initialization**
   - Initialize population $N$ based on Eq. (9)
   - Evaluate objective functions
   - Rank objective functions and find $p_g$

3. **Loop:**
   - while ($t < T_{\text{max}}$)
     - For $i = 1: N$
       - For CSO
         - Update the location of the cuckoo using Eq. (10)
       - For ICSO
         - Update the location of the cuckoo using Eq. (11)
     - End for
     - A fraction $β$ of worse solution abandoned and new ones are generated based on Eq. (9)
     - Rank objective functions and update $p_g$
     - $t = t + 1$
   - End while

4. **Print the Optimal Solution**

   **Fig. 4** – *Pseudo code of ICSO.*

5 **Simulation Study**

The simulation outcomes and discussion of the I-PD controller approach tuned by swarm optimization for production-inventory systems are given in this section. MATLAB program is utilized to conduct simulations of the controlled system and evaluate the dynamics performance. The production-inventory system that is described by (1) – (3) is used in the simulation. In this study, the following assumptions are considered: the time constant of the forecasting demand $T_a$ is one day and the time constant of the production lag $T_p$ is two days.

In addition, the model of the production-inventory system is linear. This assumption means that if demand is unmet, it can be back-ordered. On other words, if the inventory level is negative, it refers to the amount of back-order.
Besides, there is no limitation in the production capacity. Lastly, a negative order rate is allowed [12].

The performance of the I-PD controller to control the inventory level is evaluated by tuning the adjusted design parameters of the control signal that is given in (8). Two swarm optimizations (CSO and ICSO) are used for the tuning process. The Integral Time of Absolute Errors (ITAE) which is defined in (12) [32] is employed as a cost function in the optimization algorithm to evaluate the controlled system.

\[
ITAE = \int_{t=0}^{t_{\text{sim}}} t |e| \, dt ,
\]

where \( t_{\text{sim}} \) is the time of the simulation and \( e \) refers to the error as given in (6).

The ITAE index penalizes positive and negative errors equally. This means that the cost of the positive inventory (i.e., holding cost) and the cost of the negative inventory (i.e., backorder cost) are equal. The minimum ITAE means the system has a better inventory level responsiveness, in other words, has less inventory cost.

To evaluate the system, the system is subjected to a unit step input (a unit step input represents a sudden change in the demand). The parameters of the CSO and ICSO are given in Table 1.

The values of the designed gains \( k_p \), \( k_i \) and \( k_d \) of the I-PD controller based on CSO and ICSO tuning process are reported in Table 2. Fig. 5 depicts the response of the I-PD-CSO and the I-PD-ICSO controllers for a unit step input. Besides, the ITAE index, the Integral of Square Error (ISE) index as given in (13) is also used to evaluate the performance of each tuning methods.

\[
ISE = \int_{t=0}^{t_{\text{sim}}} e^2 \, dt .
\]

Table 3 shows the ITAE and ISE indices index of the system for both controllers.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CSO</td>
</tr>
<tr>
<td>Population Size (( N ))</td>
<td>25</td>
</tr>
<tr>
<td>Number of Iterations (( T_{\text{max}} ))</td>
<td>40</td>
</tr>
<tr>
<td>Probability (( \beta ))</td>
<td>0.25</td>
</tr>
<tr>
<td>Step size (( \alpha ))</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 2
Controller’s parameters of I-PD-CSO and I-PD-ICSO.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CSO</td>
</tr>
<tr>
<td>$k_p$</td>
<td>78</td>
</tr>
<tr>
<td>$k_i$</td>
<td>12</td>
</tr>
<tr>
<td>$k_d$</td>
<td>14.3</td>
</tr>
</tbody>
</table>

Table 3
Performance comparison between I-PD-CSO and I-PD-ICSO for unit step.

<table>
<thead>
<tr>
<th>Performance Index</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I-PD-CSO</td>
</tr>
<tr>
<td>$ITAE$</td>
<td>8.03</td>
</tr>
<tr>
<td>$ISE$</td>
<td>2.07</td>
</tr>
</tbody>
</table>

From Fig. 5, it can be seen that the I-PD controller for both tuning methods stabilizes the production-inventory system. However, the I-PD controller tuned by ICSO provides better response performance.

Fig. 5 – Step Response of I-PD-CSO and I-PD-ICSO.

Moreover, it can be seen from Table 3 that the value of the $ITAE$ of the system based on the I-PD-ICSO controller (5.1) is less than the value of the $ITAE$ of the system with the I-PD-CSO controller (8.03). Besides, the value of the $ISE$
of the system based on the I-PD-ICSO controller (1.52) is less than the value of the \textit{ISE} of the system with the I-PD-CSO controller (2.07). This result reveals that the I-PD-ICSO controller can achieve better performance than the I-PD-CSO controller for the production-inventory system in the case of a change in the demand following step changes.

For a more realistic business scenario, the two controlled systems are subjected to a random demand. The demand signal pattern is shown in Fig. 6. The same value of the designed gains of the I-PD controller that are obtained in the case of the unit step is used to evaluate the robustness of the two controllers under random demand. Fig. 7 depicts the inventory response of the system based on the I-PD-CSO and the I-PD-ICSO controllers. Besides, Table 4 shows the value of the ITAE and ISE indices of the system for both controllers under a random demand.

![Demand pattern](Fig. 6 - Demand pattern)

Based on Fig. 7, it can be noticed that I-PD controller for both tuning methods stabilize the system (the oscillation of the inventory level does not increase with time). However, the I-PD controller tuned by ICSO provides better response performance where the oscillation of the inventory level in the case of the I-PD-ICSO controller has less amplitude than the oscillation of the inventory level in the case of the I-PD-CSO controller.

In addition, it can be seen from Table 4 that the value of the \textit{ITAE} index of the system based on the I-PD-ICSO controller (3781) is smaller than the value of the \textit{ITAE} index of the system based on the I-PD-CSO controller (3857). These results confirm that the I-PD-ICSO controller can achieve better performance
than the I-PD-CSO controller for the production inventory system even if the change in demand is random. Besides, the value of the $ISE$ of the system based on the I-PD-ICSO controller (136) is less than the value of the $ISE$ of the system with the I-PD-CSO controller (148).

![Fig. 7 – Response of I-PD-CSO and I-PD-ICSO for random demand.](image)

**Table 4**

Performance comparison between I-PD-CSO and I-PD-ICSO for random demand.

<table>
<thead>
<tr>
<th>Performance Index</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I-PD-CSO</td>
</tr>
<tr>
<td>$ITAE$</td>
<td>3857</td>
</tr>
<tr>
<td>$ISE$</td>
<td>148</td>
</tr>
</tbody>
</table>

The aforementioned results of the two simulations demonstrate that the ICSO algorithm is capable of effectively tuning the I-PD controller to improve the responsiveness of the production-inventory system. However, these findings must be further cross-validated on other fields, such as electrical engineering, electronics, and automatic control in the future.

6 Conclusion

In this research, a single-stage single-product production-inventory system with zero desired inventory level is considered. The order quantity in this work is considered the decision variable that is needed to be determined. A modified
version of the PID controller named Integral minus Proportional Derivative (I-PD) controller is adopted to find the sophisticate order rate to control the production-inventory system. Cuckoo Search Optimization (CSO) and an improved version of the algorithm (ICSO) are proposed to tune the adjustable parameters of the I-PD controller. To ensure a realistic environment, the design procedure covers two demand scenarios. In the first one, it is assumed that the demand has a sudden change in its amplitude, whereas in the other case, it is considered that the change in the demand is random to represent more challenging demand scenario. The \textit{ITAE} and \textit{ISE} indices are used to assess the dynamics performance of the system. The simulation results show that the proposed I-PD controller for both scenarios and for both tuning methods stabilize the production-inventory system. However, the results reveal the superiority of ICSO over CSO to tune the I-PD control in terms of improving the inventory level responsiveness. This improvement has a direct impact on reducing inventory costs.

The study is limited to a linear model of a manufacturing system. Therefore, an investigation of the impact of nonlinearity in the performance of the system including production capacity constraints and/or a limitation of a non-negative order rate could be considered as future work.

7 References


