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Thermal Field Diffusion in One, Two and Three-Dimensional Half Space

Grigore A. Cividjian[1], Dumitru Broscareanu[2]

Abstract: The diffusion of suddenly occurring local high temperature in homogeneous half-infinite space is studied in the cases of one, two and three-dimensional half space. Comparison of the three cases is made. Applications of theoretically analyzed models are suggested. Errors induced by assumptions are evaluated.

Keywords: Transient and stationary thermal field, Finite element method (FEM), Numerical inverse Laplace transform, Temperature penetration depth, Characteristic length, Thermal field localization and spread, Inverse problem.

1 Introduction

Systematical, unitary analysis and graphical presentation of thermal field diffusion in homogeneous media can facilitate some inverse problems solving, as fire localization, its causes identification and various homogeneous elements refractoriness.

For instance, when the electrical short-circuit occurs the contact is rapidly melting around the arc foot and a half spherical or cylindrical isothermal surface of radius r_0 with constant temperature θ^* (equal to metal melting point e.g.) can be considered (the hot layer).

We will consider a half infinite homogenous space (x ≥ 0) with 1, 2 and 3 dimensions and c, γ, λ, specific heat, density, thermal conductivity and zero initial temperature, subjected to a step of temperature $\theta_1^*(t)$ at radius r_0 (hot layer) and constant zero temperature at infinity. For given time t after the temperature step is applied, we will define a temperature penetration depth as:

$$x_t = \sqrt{at}, \quad a = \frac{\lambda}{c\gamma}. \tag{1}$$

[1]University of Craiova, Department of Electrical Engineering, 200440, Craiova, Romania;
E-mail: [a]gcividjian@rdslink.ro
[2]University of Craiova, Department of Electrical Engineering, 200440, Craiova, Romania;
E-mail: miticabd@yahoo.com

Using the defined penetration depth, in the paper is analyzed how close to the simplest one dimensional case are the two other cases when r_0 increases.

2 One Dimensional Case

A thin half infinite homogenous rod (wire) with A cross-section and c, γ, λ, ρ, specific heat, density, thermal conductivity and electrical resistivity, carrying a constant direct current I, is considered (Fig. 1a).

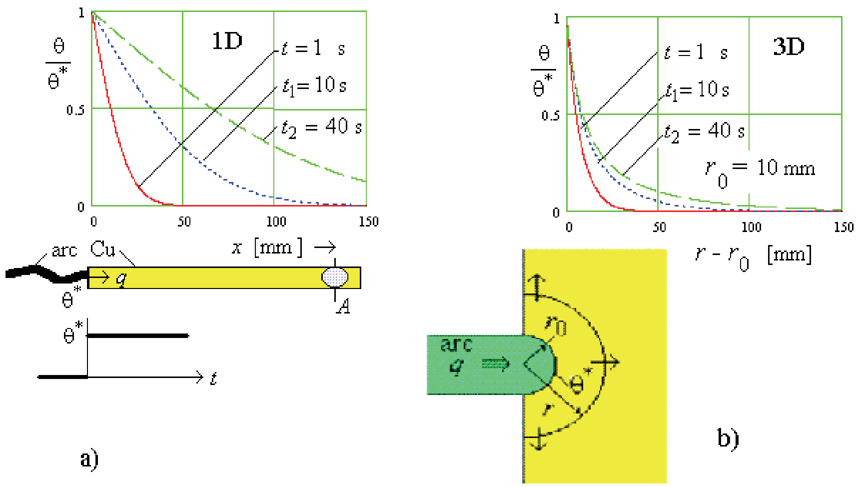

Fig. 1 – *Samples of one and three-dimensional thermal field: contact heating by electric arc.*

The heat transfer equation, which results from energy conservation, considering the Fourier and Newton laws, for relatively large difference in temperature between the solid surface and surrounding fluid area can be written as follows, for a solid volume V with the mass m, loss density p, resistivity ρ, cooling surface S and heat transfer coefficient α [1]:

$$\frac{\partial \theta}{\partial t} = a\Delta\theta - \frac{\theta}{\tau} + \frac{p}{c\gamma}, \quad \tau = \frac{\iiint\limits_V c\gamma \,\mathrm{d}v}{\oiint\limits_S \alpha \,\mathrm{d}S} = \frac{mc}{\alpha S}, \quad p = \rho j^2 . \tag{2}$$

In our case, the current density j is constant along the wire and, using the index 0 for the quantities at ambient temperature and considering the temperature coefficient of resistivity α_R, this equation becomes:

$$\frac{\partial\theta}{\partial t} = a\Delta\theta - \frac{\theta}{\tau_I} + \frac{p_0}{c\gamma}, \quad a = \frac{\lambda}{c\gamma},$$
$$\tau_I = \frac{mc}{\alpha S - \alpha_R P_0}, \quad p_0 = \rho_0 j^2, \quad P_0 = R_0 I^2, \tag{3}$$

where m and R_0 are the wire mass and resistance per unit length at ambient temperature and τ_I the equivalent local time constant, taking into account the wire material resistivity dependence on temperature α_R, for constant current.

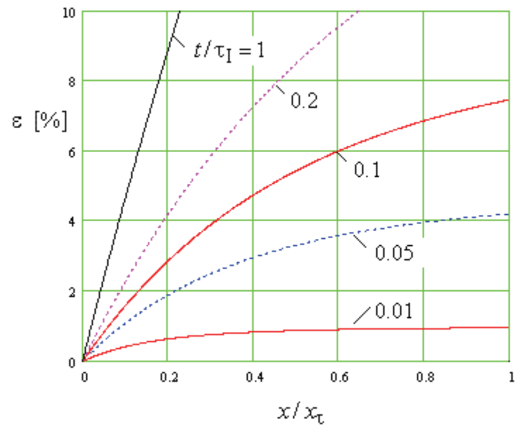

Fig. 2 – *Errors of* (11) *versus* (9).

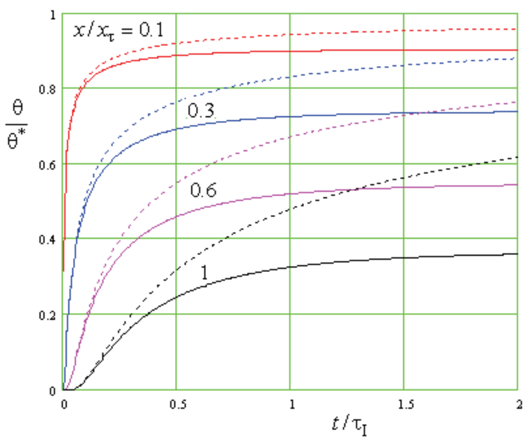

Fig. 3 – *Temperature versus time for several distances, calculated with* (9) *– solid line and* (11) *– dot.*

We will consider isothermal all the rod cross-sections and the temperature of the rod depending only on x and t. For the Laplace transform $T(x,s)$ of the temperature the following equation results from (3):

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} - \left(s + \frac{1}{\tau_\mathrm{I}}\right)\frac{T}{a} + \frac{p_0}{s\lambda} = 0. \tag{4}$$

The boundary conditions for this problem are the following:

$$T(0,s) = \frac{\theta^*}{s}, \quad T(\infty,s) < \infty. \tag{5}$$

The solution of this ordinary equation with constant coefficients is the sum of general solution of homogenous equation and the particular solution T_m of non-homogenous equation:

$$T(x,s) = T_m + \left(\frac{\theta^*}{s} - T_m\right)e^{-\beta x}, \quad \beta = \sqrt{\frac{1}{a}\left(s + \frac{1}{\tau_\mathrm{I}}\right)}, \quad T_m = \frac{p_0}{c\gamma}\frac{1}{s\left(s + \frac{1}{\tau_\mathrm{I}}\right)}. \tag{6}$$

To find the original of this expression we will apply the translation theorem of Laplace transform [2] 4.1. (5) to the following Laplace images, which can be found in [2] at 5.6 (3) and (10), in terms of error and complementary error functions (Erf and Erfc):

$$\frac{e^{-\sqrt{\alpha' s}}}{s} \quad \Leftrightarrow \quad \mathrm{Erfc}\left(\frac{1}{2}\sqrt{\frac{\alpha'}{t}}\right), \quad \mathrm{Re}(\alpha') \ge 0,$$

$$\frac{2e^{-\sqrt{\alpha' s}}}{s + \beta'} \quad \Leftrightarrow \quad e^{-\beta' t}\left[e^{-i\sqrt{\alpha'\beta'}}\,\mathrm{Erfc}\left(\frac{1}{2}\sqrt{\frac{\alpha'}{t}} - i\sqrt{\beta' t}\right) + \right. \tag{7}$$

$$\left. + e^{i\sqrt{\alpha'\beta'}}\,\mathrm{Erfc}\left(\frac{1}{2}\sqrt{\frac{\alpha'}{t}} + i\sqrt{\beta' t}\right)\right].$$

The temperature of the uniformly cooled wire, carrying a constant current I, will be:

$$\theta(x,t) = \theta_\mathrm{m}\left[1 - e^{-t/\tau_\mathrm{I}}\,\mathrm{Erf}(z)\right] +$$

$$+ \frac{\theta^* - \theta_m}{2}\cdot\left[e^{\frac{-x}{x_\tau}}\,\mathrm{Erfc}\left(z - \sqrt{\frac{t}{\tau_\mathrm{I}}}\right) + e^{\frac{x}{x_\tau}}\,\mathrm{Erfc}\left(z + \sqrt{\frac{t}{\tau_\mathrm{I}}}\right)\right], \tag{8}$$

$$\theta_m = \frac{p_0\,\tau_\mathrm{I}}{c\gamma}, \quad z = \frac{x}{2\sqrt{a t}} = \frac{x / x_\tau}{2\sqrt{t/\tau_\mathrm{I}}}, \quad x_\tau = \sqrt{a\,\tau_\mathrm{I}}.$$

2.1 Characteristic length of thermal field diffusion

The quantity $x_t = \sqrt{at}$ was called the temperature penetration depth in t seconds. It depends only on the (homogeneous) material and the considered time t. For time equal to the local thermal time constant it can be called characteristic length of thermal field diffusion and depends on dimensions, cooling conditions and material characteristics or on temperature diffusion coefficient a and local thermal time constant τ_{I}.

For $\theta_m \ll \theta^*$ (for example without current $\theta_m = 0$) and the above equation becomes:

$$\theta(x,t)\big|_{I=0} = \frac{\theta^*}{2}\cdot\left[e^{\frac{-x}{x_\tau}}\,\mathrm{Erfc}\left(z-\sqrt{\frac{t}{\tau_{\mathrm{I}}}}\right)+e^{\frac{x}{x_\tau}}\,\mathrm{Erfc}\left(z+\sqrt{\frac{t}{\tau_{\mathrm{I}}}}\right)\right]. \tag{9}$$

The square bracket from (8) or (9) can be approximated as follows:

$$\left[\mathrm{Erfc}(z)+\frac{XT}{\sqrt{\pi}}e^{-z^2}\right]\mathrm{ch}(X)-\frac{2T}{\sqrt{\pi}}e^{-z^2}\,\mathrm{sh}(X)\approx$$

$$\approx \mathrm{Erfc}(z)\,\mathrm{ch}(X)-\frac{T}{\sqrt{\pi}}e^{-z^2}\,\mathrm{sh}(X), \tag{10}$$

$$X=\frac{x}{x_\tau}<1, \quad T=\frac{t}{\tau_{\mathrm{I}}}<1.$$

Since $\tau_{\mathrm{I}} > 0$, for poor cooling conditions (small α) the time constant and the characteristic length tend to infinity and for small times the terms t/τ_{I} and x/x_τ in (7) tend to zero. For heat transfer coefficient α tending to zero the equation (8) becomes:

$$\theta(x,t)\big|_{\alpha=0} \approx \theta^*\mathrm{Erfc(z)},$$

$$z=\frac{x}{2\sqrt{at}}=\frac{x}{2x_t}. \tag{11}$$

The last equation gives up to 1% larger values of the temperature than (9), while $x < x_\tau$ and $t < 0.01\tau_{\mathrm{I}}$. The errors for several values of t/τ_{I} are given in Fig. 2.

In Figs. 3 and 4 can be seen that the two formulas (11) and (9) agree enough well up to relatively small times ($t/\tau_{\mathrm{I}} < 0.1$).

How the 1D thermal field is the less concentrated, it results from (11) that, at distance from hot later equal to penetration depth, the temperature cannot be more than $\mathrm{Erfc}(0.5)\cdot\theta^* = 0.48\,\theta^*$. So, the penetration depth at time t, x_t could be defined as the distance from hot layer at which the temperature at time t is no more than $0.48\,\theta^*$.

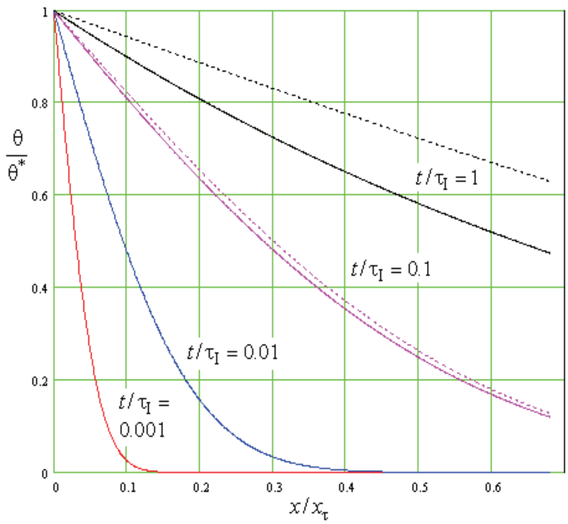

Fig. 4 – *Temperature versus distance, calculated for several times with* (9) *– solid line and* (11) *– dot.*

2.2 Thermal flux density

The thermal flux density across the wire ($q = -\lambda\nabla\theta$) results from the derivative of (8) with respect to x:

$$q = \frac{\lambda}{\sqrt{\pi a t}}\left\{\theta_\mathrm{m}\, e^{-(z^2+T)} + \frac{\theta^* - \theta_\mathrm{m}}{2}\times\left[e^{-X}\left(\sqrt{\pi T}\,\mathrm{Erfc}(z-\sqrt{T}) + e^{-(z-\sqrt{T})^2}\right) - e^{X}\left(\sqrt{\pi T}\,\mathrm{Erfc}(z+\sqrt{T}) - e^{-(z+\sqrt{T})^2}\right)\right]\right\}. \quad (12)$$

When τ_I tends to infinity X and T approach zero and the thermal flux density becomes:

$$q\big|_{\tau_I\to\infty} = \frac{\lambda\,\theta^*}{\sqrt{\pi a t}}\, e^{-z^2}. \quad (13)$$

In particular, for $x = 0$ we have:

$$q\big|_{\substack{x=0\\ \tau_I\to\infty}} = \frac{\lambda\,\theta^*}{\sqrt{\pi a t}} = \theta^*\sqrt{\frac{\lambda c\gamma}{\pi t}} \quad [\mathrm{W/m^2}]. \quad (14)$$

The thermal energy absorbed by the rod is:

$$Q = \frac{2}{\sqrt{\pi}} \theta^* c\gamma A \sqrt{a t} = \frac{2}{\sqrt{\pi}} \theta^* A \sqrt{c\gamma\lambda t} \quad [\mathrm{J}]. \tag{15}$$

3 Two Dimensional Axisymmetric Case

In this case, at constant current, the current density decreases inversely proportional to the radius r. Hence the conductor losses density drops off inversely proportional to r^2 and can be neglected. The cooling surface is very small and the time constant can be considered infinite.

Due to the symmetry, the temperature will be considered as function only of radius and time and the equation (2) becomes:

$$\frac{\partial\theta}{\partial t} = \frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r} \right) + \frac{\theta}{\tau}. \tag{16}$$

For the Laplace transform of the temperature with respect to the time $T(r,s)$ it results the following modified Bessel equation [3]:

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} + \frac{1}{x}\frac{\mathrm{d}T}{\mathrm{d}x} - \nu^2 T = 0, \qquad x = \nu r, \quad \nu = \sqrt{\frac{1}{a}\left(s + \frac{1}{\tau} \right)}. \tag{17}$$

We will consider the temperature of r_0 radius cylindrical surface at $t = 0$ instantly rising from 0 to θ^* and constant after that. As in previous case, at r infinite the temperature must rest finite one:

$$\theta(r_0,t) = \theta^* \cdot 1(t) \quad \Rightarrow \quad T(0,s) = \frac{\theta^*}{s}, \quad \theta(\infty,t) < \infty \quad \Rightarrow \quad T(\infty,s) < \infty. \tag{18}$$

The solution of this problem is expressed by modified Bessel functions of zero order:

$$T(r,s) = \frac{\theta^*}{s}\frac{\mathrm{K}_0(\nu r)}{\mathrm{K}_0(\nu r_0)}. \tag{19}$$

Unfortunately there is not closed form of the original of this expression, but, using the best located nodes [4], it can be enough exact calculated, using the equation:

$$\theta(r,t) \approx \theta^* \sum_{i=1}^{10} A_i \frac{\mathrm{K}_0\left(\frac{r}{\sqrt{a t}}\sqrt{p_i} \right)}{\mathrm{K}_0\left(\frac{r_0}{\sqrt{a t}}\sqrt{p_i} \right)}. \tag{20}$$

The coefficients A_i and the nodes p_i are calculated in [4] with 20 significant digits and approximately given near the equation (20), in the next table.

p =

1	5.225+15.73i
2	5.225-15.73i
3	8.776+11.922i
4	8.776-11.922i
5	10.934+8.41i
6	10.934-8.41i
7	12.226+5.013i
8	12.226-5.013i
9	12.838+1.666i
10	12.838-1.666i

A =

1	-10.349 +4.111i
2	-10.349 -4.111i
3	186.327 -253.322i
4	186.327 +253.322i
5	$-858.652 +2.322i\cdot10^3$
6	$-858.652 -2.322i\cdot10^3$
7	$1.552\cdot10^3 -8.44i\cdot10^3$
8	$1.552\cdot10^3 +8.44i\cdot10^3$
9	$-868.461 +1.546i\cdot10^4$
10	$-868.461 -1.546i\cdot10^4$

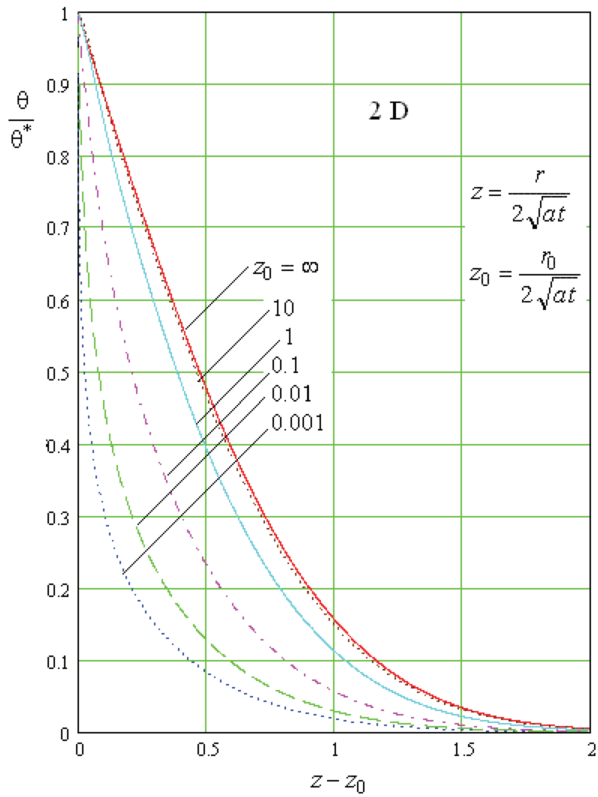

Fig. 5 – *Temperature versus the distance from hot surface for cylindrical isotherms.*

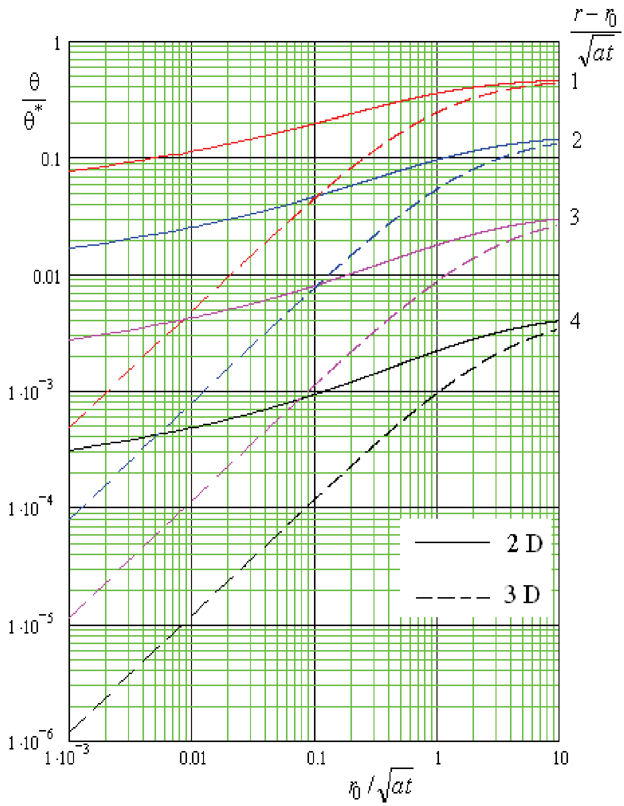

Fig. 6 – *Fraction of temperature at several* x_t *from the hot layer versus its radius* r_0/x_t *for cylindrical (solid) and spherical (dashed) case.*

3.1 Thermal field localization in 2D case

In assumed conditions the temperature θ^* will spread to all the half space at infinite time. At given time t the hot region (with temperature larger than θ) can be determined from Fig. 5, where θ/θ^* is given as function of distance from constant temperature hot surface with radius of this surface as parameter.

When r_0 increases the thermal field approaches the field in one-dimensional case. In Fig. 5 can be seen that for $r_0 > 10\sqrt{at}$ the one-dimensional case can be considered. At smaller r_0 the temperature decreases more at same distance from the hot layer. The hot region is smaller than in 1D case, the thermal field is more localized. For all r_0 the temperature falls to less than

$\text{Erfc}(1)\cdot\theta^* = 0.157\,\theta^*$ at distance larger than $2x_t$ and than $0.0047\,\theta^*$ at distance larger than $4x_t$:

$$\theta(r,t) < \begin{cases} 0.156\,\theta^*, & r - r_0 > 2\sqrt{at}, \\ 0.00468\,\theta^*, & r - r_0 > 4\sqrt{at}. \end{cases} \tag{21}$$

In Fig. 6, the fraction of temperature θ^* at distance from hot layer of several times the penetration depth is given as function of hot layer radius over the penetration depth ($2z_0$). The dashed lines are the same for three-dimensional case.

The FEM analysis of complex configurations can be limited at the domain where the temperature is higher than environmental. In function of the hot layer relative radius $x_0 = r_0/\sqrt{at}$ and the selected level of precision, the necessary distance from the hot layer can be determined in this figure. For example, if 1% of maximum temperature can be considered 0 and $x_0 = 0.2$, the considered for FEM domain must have the external border at least at $3x_t$ from the hot layer.

4 Three Dimensional Spherical Case

In this case the conductor losses density drops off with $1/r^4$ and the cooling surface is also very small.

The time constant for a homogeneous cylinder and sphere with isotherm surface and r radius are:

$$\tau_c = \frac{c\gamma r}{2\alpha}, \qquad \tau_s = \frac{c\gamma r}{3\alpha}. \tag{22}$$

As in previous case we will neglect the heating effect of the current and the cooling. With these assumptions the equation (2) for central symmetry can be written as follows:

$$\frac{\partial\theta}{\partial t} = a\,\Delta\theta = \frac{a}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right), \quad a = \frac{\lambda}{c\gamma}. \tag{23}$$

Changing the function $\theta = \theta_1/r$ the equation for θ_1 and its Laplace transform will be:

$$\frac{\partial\theta_1}{\partial t} = a\frac{\partial^2\theta_1}{\partial r^2}, \quad \frac{\mathrm{d}^2 T_1}{\mathrm{d}r^2} = \frac{s}{a}T_1. \tag{24}$$

The initial conditions of the problem being:

$$T_1(r_0, s) = \theta^* \frac{r_0}{s}, \tag{25}$$

the solution for the image will be:

$$T_1(r, s) = \theta^* \frac{r_0}{s} e^{-(r-r_0)\sqrt{\frac{s}{a}}}. \tag{26}$$

Using the equations (6) the closed form for original function can be obtained [2] and the thermal field will be given by the equation:

$$\theta(r, t) = \theta^* \frac{r_0}{r} \mathrm{Erfc}(z), \quad z = \frac{r - r_0}{2\sqrt{a t}}. \tag{27}$$

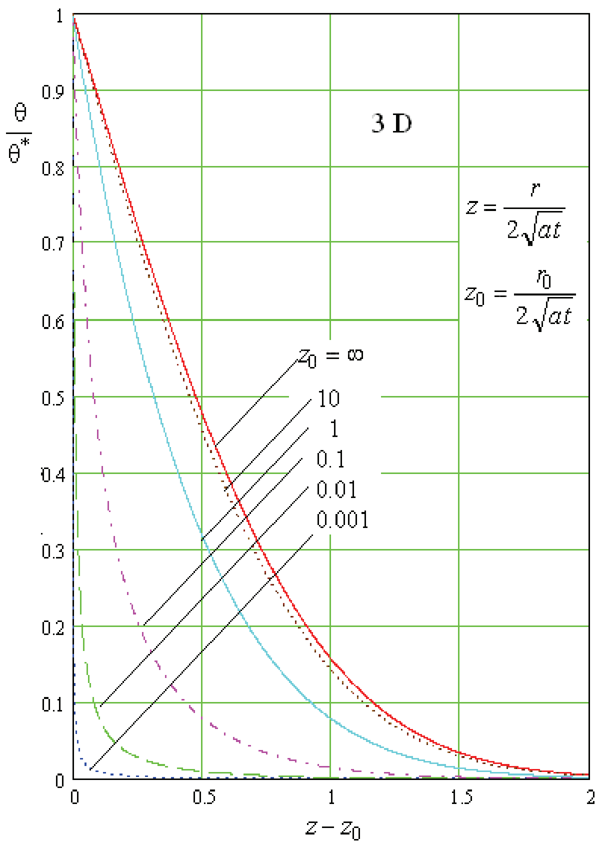

Fig. 7 – *Temperature versus the distance from hot surface for spherical isotherms.*

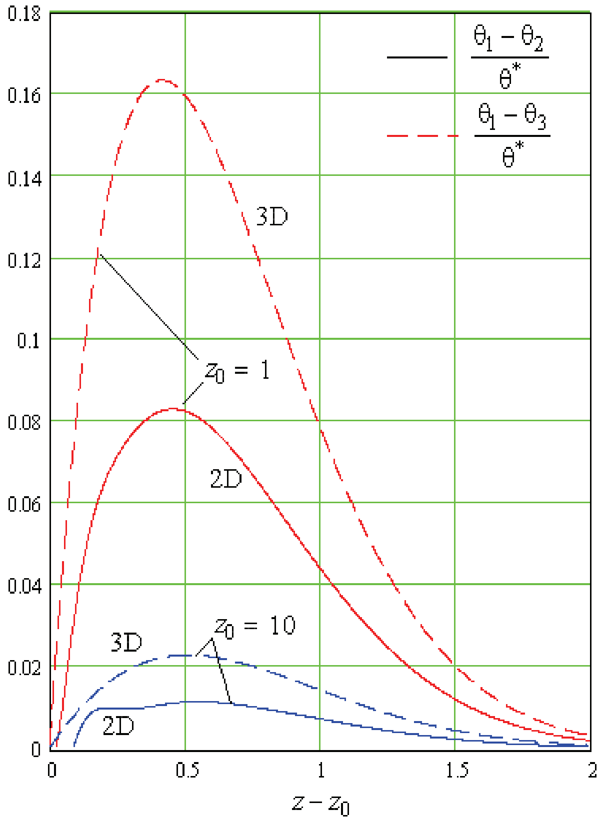

Fig. 8 – *Differences between the relative temperatures in* 1D *and* 2D (*solid*) *or* 3D (*dashed) radius versus the relative distance between layers, for two values of relative hot layer* z_0.

4.1. Thermal field localization

Comparing the Figs. 5 and 7 can be observed that the three-dimensional thermal field is more localized than the cylindrical one. In both cases, at given instant, the temperature distribution with radial distance from the hot surface is under the curve of one-dimensional case, approaching it when the radius r_0 of hot surface increases.

The difference between the temperature in one-dimensional case θ_1 and θ_3 for three-dimensional case is given by the equation resulting from (11) and (27):

$$\frac{\theta_1 - \theta_3}{\theta^*} = \frac{z - z_0}{z} \mathrm{Erfc}(z - z_0) . \tag{28}$$

In Fig. 8 the differences between the temperature in one-dimensional case θ_1 and θ_2 for two-dimensional case, respectively θ_3 for three-dimensional case are given (in relative units) as functions of the radial distance, for two values of z_0.

It can be observed that the larger differences are around $z-z_0$ equal to 0.5 and in 2D case are two times smaller than in 3D case.

4.2 Thermal flux density

In 3D case the thermal flux density results from (27):

$$q = -\lambda\nabla\theta = \theta^*\lambda\frac{r_0}{r^2}\left[\mathrm{Erfc}(z) + \frac{r\exp(-z^2)}{\sqrt{\pi a t}}\right]. \tag{29}$$

The total thermal flux entering in the half space will be, considering the hot layer area $A = 2\pi r_0^2$:

$$q_1 = -2\lambda\pi r_0^2 \left.\frac{\mathrm{d}\theta}{\mathrm{d}r}\right|_{r_0} = 2\pi r_0\lambda\theta^*\left[1 + \frac{r_0}{\sqrt{\pi a t}}\right], \tag{30}$$

and the energy:

$$Q = 2\pi r_0\lambda\theta^*\left[t + \frac{2r_0}{\sqrt{\pi a}}\sqrt{t}\right]. \tag{31}$$

5 Applications

5.1 Example 1

The short –circuit in copper conductor can last up to 10 seconds. Which minimal distance from the hot layer must be adopted for FEM analysis domain to obtain a detailed solution of thermal field with 1% of θ^* precision?

The less concentrated thermal field is the one-dimensional. The thermal penetration depth for $t = 10\,\mathrm{s}$ in copper ($a = 118\,\mathrm{mm}^2$) is

$$x_t = \sqrt{118\cdot 10} = 34.3\,\mathrm{mm}. \tag{E1}$$

From (11) it results the minimal distance from hot layer must be at least $3.62x_t = 124\,\mathrm{mm}$.

From Fig. 6 it results that for $r_0 < 0.2x_t = 6.9\,\mathrm{mm}$ in two-dimensional case and for $r_0 < x_t = 34\,\mathrm{mm}$ in three-dimensional case the distance can be reduced to $3x_t = 100\,\mathrm{mm}$. In three-dimensional case for $r_0 < 0.02x_t = 0.7\,\mathrm{mm}$ the distance can be taken equal to the penetration depth, $\sim 35\,\mathrm{mm}$.

5.2 Example 2

The distance between the contact and the copper conductor insulation is $L = 150\,\text{mm}$. How much time will pass after contact melting up to the moment when the temperature near insulation will reach 10% of θ^*?

From (11) or in Fig. 5 it can be seen that, in one-dimensional case and without cooling effect, $z - z_0$ must be 1.164. It results:

$$t = \frac{L^2}{4a(z - z_0)^2} = \frac{150^2}{4 \cdot 118 \cdot 1.164^2} = 35.2 \text{ s} . \qquad \text{(E2)}$$

5.3 Example 3

An electric arc made a $r_0 = 9\,\text{mm}$ radius circular hole in a large homogeneous steel sheet with temperature diffusion coefficient $a = 20\,\text{mm}^2$ and melting temperature $\theta^* = 1500°\text{C}$. Which temperature will be after $90\,\text{s}$ at $100\,\text{mm}$ from the hole center?

In Fig. 5 or using (20) it can be seen that in two-dimensional case, neglecting the cooling effect we will have respectively:

$$\begin{aligned} &x_t = \sqrt{a\,t} = 42 \text{ mm}, \quad z_0 = \frac{r_0}{2x_t} = 0.1, \\ &z - z_0 = \frac{100 - 9}{2 \cdot 42} = 1.07, \quad \frac{\theta}{\theta^*} \approx 0.05. \end{aligned} \qquad \text{(E3)}$$

Up to $90\,\text{s}$ the temperature will not exceed $1500 \times 0.05 = 75°\text{C}$.

5.4 Example 4

The hot surface with $r_0 = 5\,\text{cm}$ has a constant temperature θ^*. After 1 hour the temperature at $d = 20\,\text{cm}$ is $0.3\theta^*$. What is the thermal diffusivity a?

From Fig. 9 it results:

$$\left.\begin{aligned} \frac{d}{r_0} = 4 \\ \frac{\theta}{\theta^*} = 0.3 \end{aligned}\right\} \Rightarrow \quad z_0 = 0.1,$$

$$t = 3600 \text{ s}, \quad x_t = \frac{r_0}{2z_0} = \frac{5}{2 \cdot 0.1} = 25 \text{ cm}, \qquad \text{(E4)}$$

$$a = \frac{x_t^2}{t} = \frac{25^2}{3600} = 0.174 \text{ cm}^2/\text{s}.$$

The temperatures from Fig. 5 can be represented as in Fig. 9.

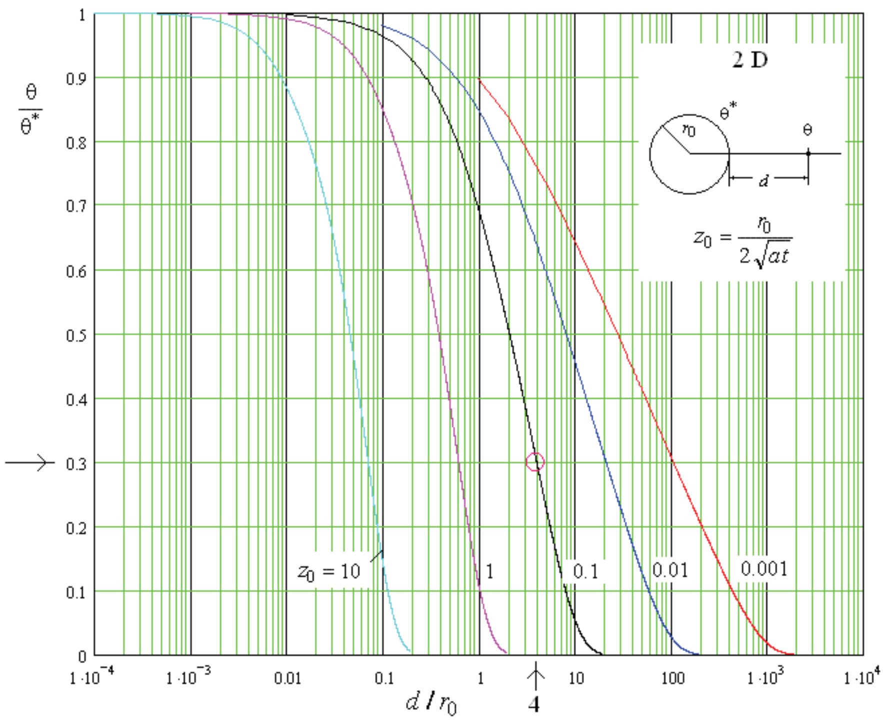

Fig. 9 – *Temperature versus the distance from hot surface for cylindrical isotherms.*

6 Conclusions

Neglecting the internal losses and surface cooling, the temperature localization and spread profile in one, two and three dimensional homogeneous space is similar if the relative distance $z - z_0$ from the hot layer is taken as variable and the hot layer relative radius z_0 as parameter.

In one-dimensional case the thermal field diffusion is given by equation (8) for the rod with current and equation (9) for the rod without current.

For negligible cooling effect ($\alpha \approx 0$) the (11) can be used. It gives up to 1% larger than exact values of the temperature, while $x < x_\tau$ and $t < 0.01\tau_I$. The corresponding errors are given in Figs. 2, 3 and 4.

For the 2D case (without current) a new approximate formula (20) is proposed.

For 3D case without current the exact formula (27) can be used.

In Figs. 5 and 7 can be seen that in the 2D case the temperature decreases faster with the distance than in the one-dimensional case and in 3D case it decreases faster than in the 2D case.

The 3D thermal field is the most localized. In both cases 2D and 3D the temperature profile at given time approaches the profile of one-dimensional case when the radius of constant temperature hot radius increases. The larger is z_0, the closer is the thermal field to the one-dimensional case. For $z_0 > 10$ (or $r_0 > 20x_t$), the one-dimensional case can be considered with an error smaller than 2.3% for 2D case and two times smaller error for 3D case. For $z_0 > 1$ the errors are respectively 16.6 % and 8.3 % (Fig. 8).

7 Acknowledgement

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