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Performance Analysis of EGC Combining over Correlated Nakagami-*m* Fading Channels

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Abstract: In this paper, performance analysis of diversity technique with equal gain combining method (EGC) with two branches for the detection of signals in wireless communication systems is presented. In the following analysis, it is assumed that the fading via channels is Nakagami-m correlated. The first order statistical characteristics of the system are analysed. Useful formulae for the probability density function (pdf) and cumulative distribution function (cdf) of EGC output SIR are derived, and the effects of the fading severity on the output signal are observed.

Keywords: Nakagami-*m* distribution, Correlated fading; EGC combining.

1 Introduction

System performances of wireless communications could be seriosly degraded by multipath fading. Therefore, various techniques for reducing fading effect are used [1]. The goal of diversity techniques is to upgrade transmission reliability without increasing transmission power and bandwidth and to increase channel capacity. Space diversity is an efficient method for improvement system's quality of service (QoS) when multiple receiver antennas are used [2]. There are several combining techniques that can be performed, depending on complexity restriction put on the communication system and amount of channel state information available at the receiver.

Diversity techniques for reducing the fading effects are better if the signals are independent, that is, if there is no correlation between the system branches. Most of the papers assume independent fading between the reception branches [3 – 5]. However, since the distance between antennas is small, it is realistic to take into consideration the signal correlation [5].

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It has been found experimentally, that the Nakagami-m distribution offers a better fit for a wider range of fading conditions in wireless communications [6, 7]. Several correlation models have been proposed and used in the literature for evaluating performance of diversity systems. The constant correlation model corresponds to a scenario with closely placed diversity antennas and circular symmetric antenna arrays [8 – 11].

An analysis of signal combining for Nakagami-*m* distributed with constant correlation model of fading has been given [12], but assuming total independence between interferences received on any pair of inputs of the combiner. In more general case, the arbitrary correlation is present between the signals and interferences.

In this paper, analytical study of dual branch EGC combining involving correlated Nakagami-*m* fading with arbitrary correlation parameter between branches is analysed.

We assume that two signals undergoing effects of fading channels have Nakagami-*m* joint probability density function. At the output of the EGC combiner, signal is equal to the sum of the two input signals. To find that sum, the input signals have to be put in phase which. EGC combining provides good results but, since it could be complicated for implementation.

2 System Model

EGC diversity combining provides intermediate solution in terms of performance and implementation complexity. In EGC receiver, each signal branch is multiplied by a complex weight and added up. Those complex weights are actually a phase corrections that provide the signal amplitudes to add up coherently, while noise is added incoherently. Each branch is real amplitude weighted with the same factor, irrespective of the signal amplitude.

In this paper, using convenient transformation of the joint probability density function, probability density function of the signal at the combiner output has been derived. Integration of this signal, provides the cumulative distribution function to be obtained. Also, characteristic function of the output signal and n-th order moment at the output of the EGC combiner, have been calculated. For $n = 1$, signal output average is presented, and for $n = 2$, average of the output signal quadratic value could be obtained, which is used for finding the variance of the output signal.

3 First Order Statistics

The performance of the two branches EGC can be carried out by considering, as in [13], the insufficient antennae spacing, both desired and interfering signal envelopes experience correlative Nakagami-*m* fading with corresponding joint distributions. We are considering constant correlation Nakagmi-*m* model of distribution.

The power correlation coefficient ρ for desired signal is defined as $cov(R_i^2, R_j^2)/(var(R_i^2)var(R_j^2))^{1/2}$ We are assuming arbitary correlation coefficients between fading signals, because correlation coefficients depend on the arrival angles of the contribution with the broadside directions of antennas.

In a two-branches EGC diversity system, at the EGC combiner input, the two signals are represented as two random variables.The joint probability density function for the two signals r_1 and r_2 is given by [5]:

$$p_{r_1r_2}\left(r_1r_2\right)=\frac{4\left(r_1r_2\right)^m e^{-\frac{r_1^2+r_2^2}{\Omega^2(1-\rho)}}}{\Gamma(m)\Omega^2\left(1-\rho\right)\left(y_1\sqrt{\rho}\right)^{m-1}}I_{m-1}\left(\frac{2\sqrt{\rho}r_1r_2}{\Omega\left(1-\rho\right)}\right). \tag{1}$$

Here, *m* represents the fading severity parameter for the desired signal. For the special case of *m*=1 we can evaluate expression for pdf for the Rayleigh signal. $I_{m-1}()$ represents the modified Bessel function of the first kind [12], while Ω is the average signal power. Γ() represents the well-known Gamma function [14].

Cumulative distribution function for two Nakagami-m random variables is obtained by integration of their joint probability density function. The closed form of such integration is performed by developing the Bessell function from the Nakagami-m distribution expression into the series and by integrating the term by term.

Cumulative distribution function of the two random variables r_1 and r_2 is obtained by integrating the joint pdf from the expression (1):

$$\begin{aligned}F_{r_1r_2}\left(r_1r_2\right)&=\int_0^{r_1}\mathrm{d}x_1\int_0^{r_2}\mathrm{d}x_2 p_{r_1r_2}\left(x_1x_2\right)=\\
&=\int_0^{r_1}\mathrm{d}x_1\int_0^{r_2}\mathrm{d}x_2\frac{4\left(x_1x_2\right)^m\exp\left(-\frac{x_1^2+x_2^2}{\Omega^2\left(1-\rho\right)}\right)}{\Gamma(m)\Omega^2\left(1-\rho\right)\left(y_1\sqrt{\rho}\right)^{m-1}}I_{m-1}\left(\frac{2\sqrt{\rho}x_1x_2}{\Omega\left(1-\rho\right)}\right)=\\
&=\int_0^{r_1}\mathrm{d}x_1\int_0^{r_2}\mathrm{d}x_2 Bx_1^m x_2^m\cdot\exp\left(-\alpha\left(x_1^2+x_2^2\right)\right)I_{m-1}\left(\beta x_1x_2\right)=\\
&=B\sum_{i=0}^{\infty}\frac{\beta^{2i+m-1}}{2^{2i+m-1}i!\Gamma\left(i+m\right)}\frac{1}{2\alpha^{i+m}}\frac{1}{2\alpha^{i+m}}\gamma\left(i+m,\alpha r_1\right)\gamma\left(i+m,\alpha r_2\right),\end{aligned} \tag{2}$$

with coefficients B and β and α_1 defined as:

$$B=\frac{4}{\Gamma(m)\Omega^2\left(1-\rho\right)\left(\Omega\sqrt{\rho}\right)^{m-1}};\quad \alpha=\frac{1}{\Omega^2\left(1-\rho\right)};\quad \beta=\frac{2\sqrt{\rho}}{\Omega\left(1-\rho\right)}, \tag{3}$$

while $\gamma(a,x)$ denoting the lower incomplete Gamma function [12].

Characteristic function of the r_1 and r_2 now can be presented as:

$$
\begin{aligned}
&M_{s_1 s_2}\left(s_1 s_2\right)=\int_0^{\infty} dr_1 \int_0^{\infty} dr_2 e^{r_1 s_1+r_2 s_2}\cdot p_{r_1 r_2}\left(x_1 x_2\right)= \\
&=\int_0^{\infty} dr_1 \int_0^{\infty} dr_2 e^{r_1 s_1+r_2 s_2} B\, r_1^{\,m}\, r_2^{\,m} e^{-\alpha\left(r_1^2+r_2^2\right)} I_{m-1}\left(\beta r_1 r_2\right)= \\
&=B\int_0^{\infty} dr_1 \int_0^{\infty} dr_2 e^{r_1 s_1+r_2 s_2} B\, r_1^{\,m}\, r_2^{\,m} e^{-\alpha\left(r_1^2+r_2^2\right)} \sum_{i=0}^{\infty} \frac{\beta^{2i+m-1}\, r_1^{\,2i+m-1}\, r_2^{\,2i+m-1}}{2^{2i+m-1} i!\,\Gamma(i+m)}= \\
&=B \sum_{i=0}^{\infty} \frac{\beta^{2i+m-1}}{2^{2i+m-1} i!\,\Gamma(i+m)} \int_0^{\infty} r_1^{\,2i+m-1} e^{-\alpha r_1^2} e^{r_1 s_1}\, \mathrm{d} r_1 \int_0^{\infty} r_2^{\,2i+2m-1} e^{-\alpha r_2^2} e^{r_2 s_2}\, \mathrm{d} r_2= \\
&=B \sum_{i=0}^{\infty} \frac{\beta^{2i+m-1}}{2^{2i+m-1} i!\,\Gamma(i+m)} \sum_{i_1=0}^{\infty} \frac{s_1^{\,i_1}}{i_1!} \sum_{i_2=0}^{\infty} \frac{s_2^{\,i_2}}{i_2!} \cdot \frac{1}{2\alpha^{i+m+i_1/2}}\cdot \\
&\cdot \frac{1}{2\alpha^{i+m+i_2/2}} \Gamma\left(i+m+i_1/2\right) \Gamma\left(i+m+i_2/2\right).
\end{aligned}
\tag{4}
$$

Joint moment of the r_1 and r_2 now can be presented as:

$$
\begin{aligned}
&M_{pq}=\overline{r_1^{\,p} r_2^{\,q}}=\int_0^{\infty} \mathrm{d} r_1 \int_0^{\infty} \mathrm{d} r_2 r_1^{\,p} r_2^{\,q} p_{r_1 r_2}\left(r_1 r_2\right)= \\
&=\int_0^{\infty} \mathrm{d} r_1 \int_0^{\infty} \mathrm{d} r_2 r_1^{\,p} r_2^{\,q} B r_1^{\,m} r_2^{\,m} e^{-\alpha_1\left(r_1^2+r_2^2\right)} I_{m-1}\left(\beta r_1 r_2\right)= \\
&=B\int_0^{\infty} \mathrm{d} r_1 \int_0^{\infty} \mathrm{d} r_2 r_1^{\,p+m} r_2^{\,q+m} e^{-\alpha_1\left(r_1^2+r_2^2\right)} \sum_{i=0}^{\infty} \frac{\beta^{2i+m-1} r_1^{\,2i+m-1} r_2^{\,2i+m-1}}{2^{2i+m-1} i!\,\Gamma(i+m)}= \\
&=B \sum_{i=0}^{\infty} \frac{\beta^{2i+m-1}}{2^{2i+m-1} i!\,\Gamma(i+m)} \int_0^{\infty} r_1^{\,p+2m+2i-1} e^{-\alpha_1 r_1^2}\, \mathrm{d} r_1 \int_0^{\infty} r_2^{\,p+2m+2i-1} e^{-\alpha_1 r_2^2}\, \mathrm{d} r_2= \\
&=B \sum_{i=0}^{\infty} \frac{\beta^{2i+m-1}}{2^{2i+m-1} i!\,\Gamma(i+m)} \cdot \frac{1}{2\alpha^{i+m+p/2}}\cdot \\
&\Gamma\left(i+m+p/2\right) \frac{1}{2\alpha^{i+m+q/2}} \Gamma\left(i+m+q/2\right).
\end{aligned}
\tag{5}
$$

At the output of the EGC combiner, the two signals from are added: $r = r_1 + r_2$, therefore: $r_1 = r - r_2$. Probability density function of the sum of the signals r_1 and r_2 is:

$$
\begin{aligned}
p_r(r) &= \int_0^r p_{r_1 r_2}(r-r_2, r_2)\,\mathrm{d}r_2 = \\
&= \int_0^r B(r-r_2)^m r_2^m e^{-\alpha\left((r-r_2)^2+r_2^2\right)} I_{m-1}\left[\beta(r-r_2)r_2\right]\mathrm{d}r_2 = \\
&= B\int_0^r (r-r_2)^m r_2^m e^{-\alpha\left((r-r_2)^2+r_2^2\right)} \sum_{i=0}^{\infty}\frac{\beta^{2i+m-1}(r-r_2)^{2i+m-1} r_2^{2i+m-1}}{2^{2i+m-1} i!\Gamma(i+m)}\mathrm{d}r_2 = \\
&= B\sum_{i=0}^{\infty}\frac{\beta^{2i+m-1}}{2^{2i+m-1} i!\Gamma(i+m)}\int_0^r (r-r_2)^{2i+2m-1} r_2^{2i+2m-1} e^{-\alpha\left[(r-r_2)^2+r_2^2\right]}\mathrm{d}r_2 = \\
&= B\sum_{i=0}^{\infty}\frac{\beta^{2i+m-1}}{2^{2i+m-1} i!\Gamma(i+m)}\sum_{k=0}^{2i+2m-1}\binom{2i+2m-1}{k} r^{2i+2m-1-k} e^{-\alpha r^2}\cdot \\
&\quad \cdot\sum_{j=0}^{\infty}\frac{\alpha^j 2^j r^j}{j!}\frac{1}{2|2\alpha|^{i+m+k/2+i/2}}\gamma\left(i+m+k/2+i/2, 2\alpha r\right) = \\
&= B\sum_{i=0}^{\infty} B_1(i,m)\cdot\sum_{k=0}^{2i+2m-1} B_2(i,m,k)\cdot\sum_{j=0}^{\infty} B_3(j)\cdot r^{2i+2m-1-k} e^{-\alpha r^2} B_4(i,j,k,m,r).
\end{aligned}
\tag{6}
$$

Characteristic function of the output signal is:

$$
\begin{aligned}
M_r(s) &= M_{r_1 r_2}(s,s) = B\sum_{i=0}^{\infty}\frac{\beta^{2i+m-1}}{2^{2i+m-1} i!\Gamma(i+m)}\sum_{i_1=0}^{\infty}\frac{s^{i_1}}{i_1!}\sum_{i_1=0}^{\infty}\frac{s^{i_2}}{i_2!} \\
&\frac{1}{2\alpha^{i+m+i_1/2}}\Gamma(i+m+i_1/2)\frac{1}{2\alpha^{i+m+i_2/2}}\cdot\Gamma(i+m+i_2/2)
\end{aligned}
\tag{7}
$$

Moment of the *n*-th order of the output signal is:

$$
\begin{aligned}
m_n &= \overline{r^n} = \overline{(r_1+r_2)^n} = \sum_{k=0}^{\infty}\binom{n}{k}\overline{r^{n-k} r_2^k} = \\
&= \sum_{k=0}^{\infty}\binom{n}{k} B\sum_{i=0}^{\infty}\frac{\beta^{2i+m-1}}{2^{2i+m-1} i!\Gamma(i+m)} \\
&\frac{1}{2\alpha^{i+m+\frac{(n-k)}{2}}}\Gamma\left(i+m+\frac{(n-k)}{2}\right)\frac{1}{2\alpha^{i+m+k/2}}\Gamma(i+m+k/2).
\end{aligned}
\tag{8}
$$

4 Numerical Results

In the Fig. 1, the probability density functions for different values of fading severity parameter *m*, for dual EGC combining over Nakagami-*m* channels, are presented.

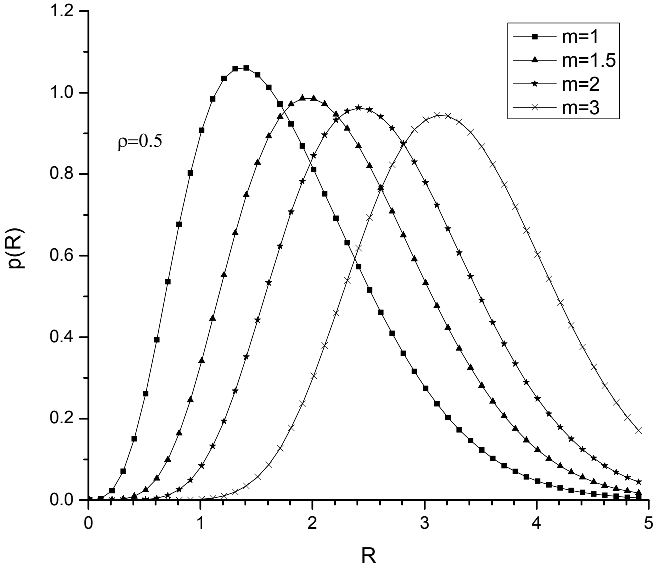

Fig. 1 – PDF *for different values of R and power correlation coefficient* ρ = 0.5.

Fig. 2 shows the output moments for different values of the severity parameter m and $\rho = 0.5$, while Fig. 3 shows the output moments for various values of correlation parameter ρ.

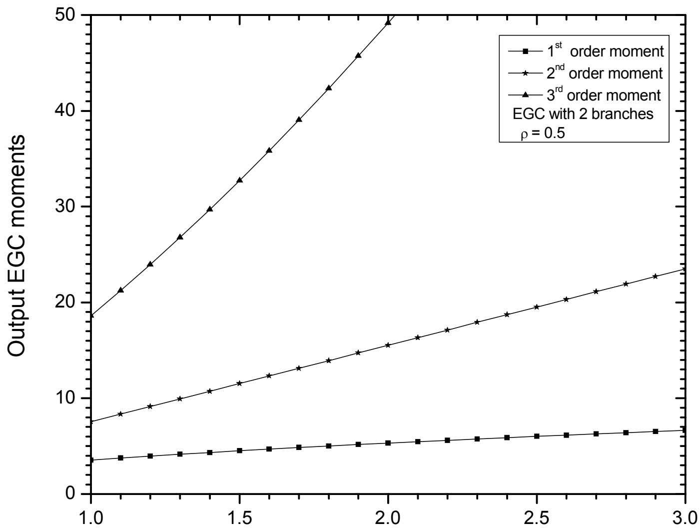

Fig. 2 – *First, second, and third output moment for different values of the the fading severity parameter m.*

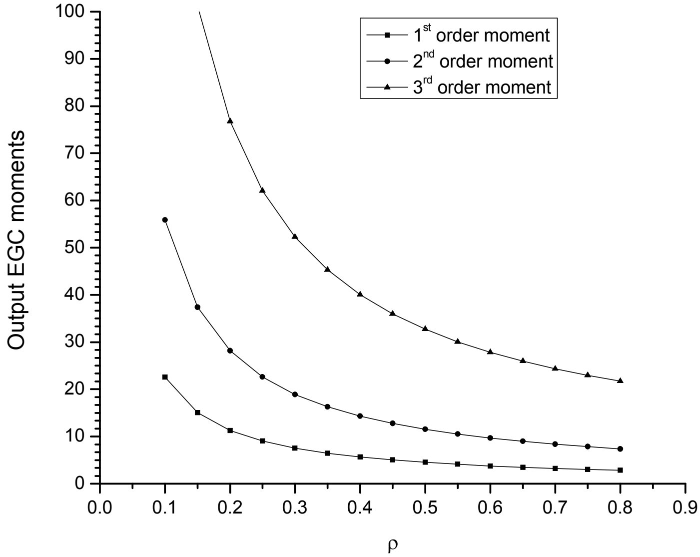

Fig. 3 – *First, second, and third output moment for different values of the power correlation coefficient* ρ.

Moment of the first order shows very slow increase, while moment of the second order increase much faster with increase of parameter m. Moment of the third order grows fast from the very beginning, that is, even for the small values of m. Moment shows an obvious decrease with the increase of the value of the correlation coefficient ρ. Decrease is more intensive for the greater order moments.

5 Conclusion

In this paper, performance analysis of EGC, operating over correlated Nakagami-*m* fading channels was presented. The proposed theoretical analysis is simple, and compared to other analysis, this one provides acceptable solution in terms of performance and implementation complexity. The useful form of formulae for pdf and cdf of EGC output SIR, were derived. Using these formulae the effects of the fading severity and the level of correlation to the output EGC moments, were observed.

6 References

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