SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 23, No. 1, February 2026, 163-179

UDC: 537.623:621.318.34 **DOI: https://doi.org/10.2298/SJEE2601163J**

Review paper

Reversibility in Some Hysteresis Models: A Brief Review

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Abstract: Electrical engineers and solid state physicists are interested in modeling hysteresis. In the present short review paper we point out the necessity to take into account the effect of reversible magnetization processes in the descriptions. We discuss some features of the Preisach, Stoner-Wohlfarth, Jiles-Atherton, Grucad and T(x) models in this context.

Keywords: Ferromagnetic hysteresis, Modeling, Reversible and irreversible magnetization processes.

1 Introduction

Ferromagnetic hysteresis is a phenomenon, which is of interest to the designers of magnetic circuits as well as to solid state physicists. Among its numerous analytical descriptions, in our opinion some models have attracted a particular attention of engineering and scientific communities, namely:

– the Preisach model (1935), subsequently scrutinized by Mayergoyz [1, 2];
– the Stoner-Wohlfarth model (S-W) (1948) [3];
– the formalism developed by Jiles and Atherton [4], as well as its thermodynamically-consistent modification advanced by the Brazilian research group GRUCAD [5];
– the mathematical $T(x)$ description [6].

In the subsequent part of the manuscript we focus on scalar versions of the aforementioned models (apart from the Stoner-Wohlfarth description, which is inherently vectorial in its nature). Moreover, we add some extra information on the application scope for the considered descriptions.

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2 Review of Hysteresis Models

The Preisach model is a purely phenomenological bottom-up approach, based on the assumption that the hysteresis curves are constructed from summation of weighted contributions from the so-called hysterons, which possess rectangular relay-like characteristics, what can be written as

$$M(t) = \Gamma(H(t)) = \iint_{a \geq b} \varphi(\alpha,\beta)\mu(\alpha,\beta)H(t)\,\mathrm{d}\alpha\,\mathrm{d}\beta\,. \tag{1}$$

The basic idea of the description is depicted in Fig. 1.

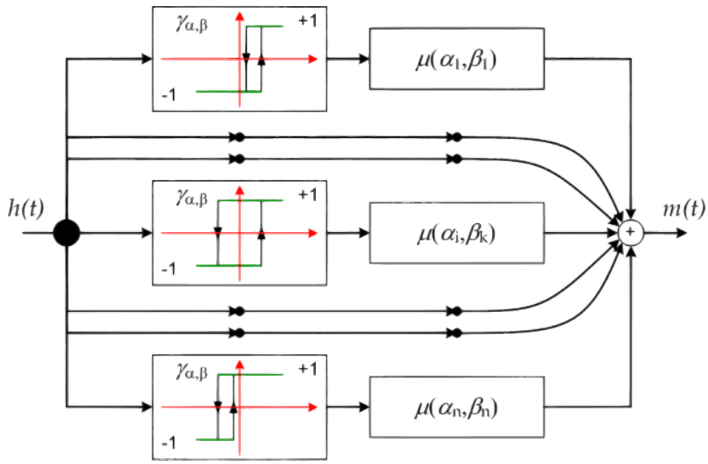

Fig. 1 – *The concept of the classical Preisach model (CPM).*

The Preisach model is a general purpose tool, it is used both by engineers and physicists. Most of research works focus on the identification of the hysteron distributions. As soon as an appropriate distribution function is chosen, the model may be treated as a "black box", capable of predicting complicated magnetization cycles. An important feature of this description is that for arbitrary magnetization patterns there is no need to update the values of its parameters (contrary e.g. to the Jiles-Atherton model).

It is easily noticeable that in the original formulation the field strength *H* is the model input and magnetization *M* is its output. However in electrical engineering it is preferable to work with flux density *B* or magnetization *M* as the input variables. There exist some approaches to "invert" the model, however they might be difficult to implement or they are demanding from the point of computation time [7, 8].

The usefulness and the wide-spread use of inverse hysteresis models in electrical engineering are due to the following facts:

- the measurements carried out in accordance with international standards IEC 60404 assume that magnetization (polarization) waveform is controlled (the so-called form factor of the secondary voltage is approximately 1.11, what implies the sine M waveform);
- electrical engineers working on design of magnetic circuits often work with Finite Element Method codes; for 2D scenarios it is common to avail of magnetic vector potential $\boldsymbol{A}$ and in the subsequent computations B is computed as $\boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A}$. For soft magnetic materials in the first approx.- imation it is possible to neglect the difference between B/μ_0 and M, thus magnetic flux density is known ahead of magnetic field strength H, which is subsequently computed from the inverse hysteresis model or its local linearization.

The model developer did not mention in his original paper that the description should take into account reversible magnetization processes. However, I. D. Mayergoyz mentioned in his monograph that the classical Preisach model exhibits some peculiar features, such as zero dynamic susceptibility $\mathrm{d}M/\mathrm{d}H$ after field reversals (sudden changes of direction of the applied magnetic field) [2]. In order to address the issue some improvements of the original model were introduced. The first approach was advanced by Gy. Kádár, who introduced a magnetization-dependent factor by which the sum of reversible and irreversible differential susceptibilities was multiplied in order to yield the total differential susceptibility [9, 10]. Another possibility was examined by E. Della Torre, who introduced a family of the so-called moving Preisach models [11]. The idea behind the moving model was to modify the distribution function in such a way that its peak moved with the magnetization of the medium. In other words the so-called effective field (accounting for interactions between individual magnetic moments, this concept is well known to those familiar with the Jiles-Atherton model) is the driving force for the moving models.

However, in the present paper we shall discuss the Kádár's concept in more detail, since it served as the foundation for some extensions of other hysteresis models, to mention [12, 13], moreover it was easier than the moving Preisach approach. The relationships between the aforementioned model extensions were discussed in the literature [14, 15].

According to Gy. Kádár, total differential susceptibility may be written as

$$\frac{\mathrm{d}M}{\mathrm{d}H} = R(m)\left(\beta + \frac{\mathrm{d}M_{\mathrm{irr}}}{\mathrm{d}H}\right), \tag{2}$$

where magnetization dependent function $R(m)$ [2] may be assumed in the first approximation as parabolic with respect to the reduced magnetization, $R(m) = 1 - m^2$. A similar concept was postulated already in 1921 by R. Gans [16]. In a subsequent publication Kádár and Szabó suggested that the shape of the function might depend on the assumed value of angular quantum number J appearing in the Brillouin function, which could describe the so-called anhysteretic curve, thus the proper choice of the functional dependence may be related to anisotropy of the considered magnetic material, Fig. 2.

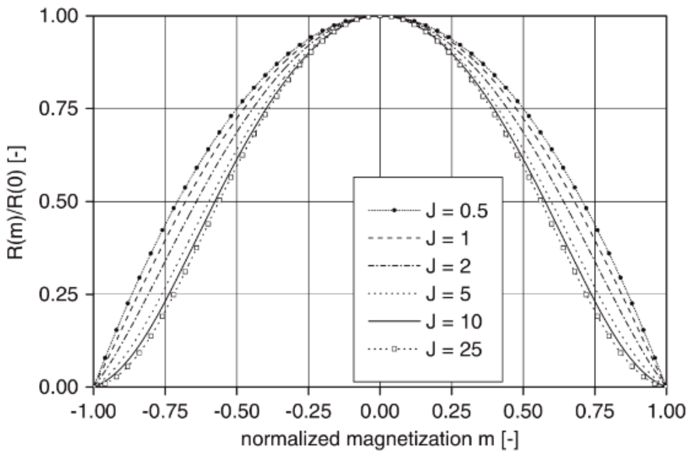

Fig. 2 – *Different shapes of the R(m) function in dependence on the value of the angular quantum number* [17].

The first term in the parentheses is a constant whose value may be recovered from the empirical Rayleigh relationship [18]. The second, irreversible term is computed from a scalar hysteresis model. Let us notice that the expression (2) may be analytically integrated, the approach has been referred to in the literature as the state-independent hypothesis [19]. The generalized form (2) was applied successfully to the Jiles-Atherton and $T(x)$ models [12, 13, 20], improving their accuracy as far as the description of minor loops was concerned.

The Stoner-Wohlfarth description is unique in the sense that a single relationship related to the energy balance equation for a prolate ellipsoid describes both irreversible and reversible magnetization processes. According to some physicists it is the only description of hysteresis loops which may be interpreted in terms of underlying physics [21]. Hysteresis curves are obtained as the result of competition between magnetic anisotropy and coupling with the

[2] $m = M/M_s$, where M_s is saturation magnetization.

applied field. In dependence on the angle θ between the easy magnetization direction and the applied field different shapes of magnetization curves occur as the result of minimizing the energy balance equation, Fig. 3.

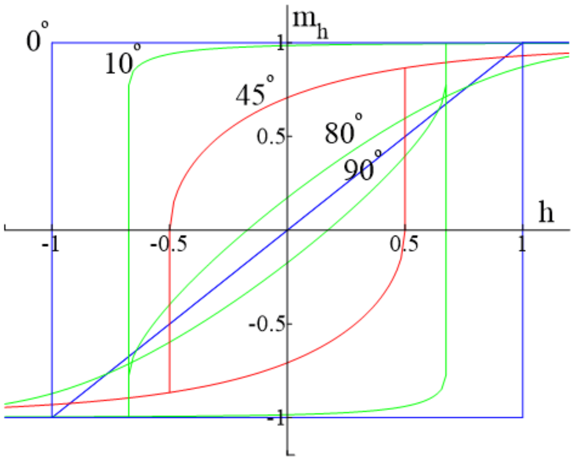

Fig. 3 – *Some magnetization curves predicted by the Stoner-Wohlfarth model for different angles between the field and the applied easy axis* [22].

Let us notice that for two extreme cases (angle θ equal to 0 or 90 degrees) the magnetization curve becomes either a relay-like rectangular dependence or a broken line. The first case corresponds to the purely irreversible material response, the second one – to purely reversible one, since the straight line segment does not exhibit hysteresis.

The Stoner-Wolfarth formalism gained a wide recognition among solid state physicists, in particular those working on thin film technologies. However, its usefulness in electrical engineering is somewhat limited since this description belongs explicitly to the group of "forward" hysteresis models, thus the applied field strength *H* is the model input and magnetization *M* is its output.

Since its introduction in the eighties of the last century the Jiles-Atherton (JA) description has attracted much attention of the scientific community due to its relatively uncomplicated mathematical structure, possibility to introduce the effects of external stimuli (e.g. applied stress, eddy currents) through the "effective field" as well as a low number of model parameters (to some of them physical meaning was attributed). The JA formalism is thus, aside from the Preisach model, a "favorite" tool for engineers, since nowadays it is straightforward to solve sets of ordinary differential equations using computer software [23]. However some of model users forget about the simplified nature of the developed description and have the tendency to use it as a Swiss army knife for scenarios for which this model in not appropriate (we mean here e.g. some attempts to use this description in computations of local variations of magnetic quantities based on diffusion equation). It should be remembered that this description is an example of "top-down" approach, which describes the averaged material response for a given sample, thus its correctness on the global scale does not imply the possibility to apply it on a local scale, e.g., at a given depth of the sample.

In the most cited "landmark" paper the JA model developers introduced the summation of irreversible and reversible magnetization components [4]. The model parameter c played the role of a weight. An interpretation for this and other parameters was provided in the literature [24]. The dimensionless parameter c "…gives a measure of the relative magnitude of the reversible magnetization contribution to the total magnetization, and is defined by the expression $\chi'_{\mathrm{in}} = c\chi'_{\mathrm{an}}$". The quantities χ'_{in} and χ'_{an} denote the differential slopes of the initial and the anhysteretic magnetization curves at the origin of the $M - H$ plane. Theoretically the definition should work fine, however these quantities are hard to be measured precisely, moreover some authors argued that the contribution of reversible magnetization should be a function of excitation level [25]. What is interesting, even in the publications of model developers one can find some statements, which may suggest that this model parameter was introduced incorrectly. In his textbook D. C. Jiles wrote [26]:

"There is of course also a reversible component of magnetization due to reversible domain wall bowing, reversible translation and reversible rotation. In the model this has the form

$$M_{\mathrm{rev}} = c\left(M_{\mathrm{an}} - M_{\mathrm{irr}}\right)$$

and since we must have either reversible or irreversible changes in magnetization, the total magnetization M_{tot} is given by

$$M_{\mathrm{tot}} = M_{\mathrm{rev}} + M_{\mathrm{irr}}.$$

In fact, the above equation is not really very helpful since magnetization changes which begin as reversible can become locked in and end up as irreversible.

A much more useful expression however is the expression for the change in magnetization with field. In this case we are more justified in distinguishing between a reversible component of susceptibility and an irreversible component of susceptibility

$$\mathrm{d}M_{\mathrm{tot}} / \mathrm{d}H = \mathrm{d}M_{\mathrm{rev}} / \mathrm{d}H + \mathrm{d}M_{\mathrm{irr}} / \mathrm{d}H\text{"}$$

whereas D.L. Atherton and M. Schönbächler wrote [27]: "(...) the reversible magnetization component is independent of field or magnetic history and is a function of magnetization only".

Let us notice that the above-given statements are consistent with the modified description introduced in [12], in which one of the equations is consistent with (1). However, for the "standard" model formulation the assumption of constant value of c either in the relationship

$$M_{\mathrm{rev}} = c\left(M_{\mathrm{an}} - M_{\mathrm{irr}}\right) \quad \text{or} \quad M_{\mathrm{rev}} = c\left(M_{\mathrm{an}} - M\right), \tag{3}$$

where $M = M_{\mathrm{rev}} + M_{\mathrm{irr}}$ implies the correctness of equivalent relationships for differential susceptibilities and *vice versa*, thus the above-given statements cannot be true.

Apart from the issue whether separation of total magnetization into irreversible and reversible components is justified there is yet another problem related to the description of reversible processes within the Jiles-Atherton framework. This is related to a peculiar model behavior after sudden field reversals, fragments of modelled quasi-static curves exhibit negative dynamic susceptibilities, Fig. 4.

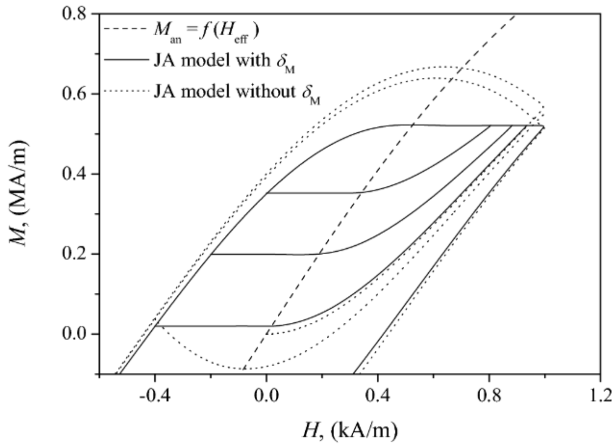

Fig. 4 – *Simulated anhysteretic curve (dashed line) and some magnetization curves without (dotted line) and with (solid line) the delta M term.*

In order to suppress the fragments of magnetization curves with negative susceptibilities a "patch" was introduced into JA model by J. H. Deane in the form of a pseudo-parameter δ_{M} defined as [28]:

$$\delta_{\mathrm{M}} = \begin{cases} 0: & \mathrm{d}H/\mathrm{d}t < 0 \ \text{and} \ M_{\mathrm{an}} - M_{\mathrm{irr}} > 0; \\ 0: & \mathrm{d}H/\mathrm{d}t > 0 \ \text{and} \ M_{\mathrm{an}} - M_{\mathrm{irr}} < 0; \\ 1: & \text{in all other cases.} \end{cases} \tag{4}$$

After the introduction of this pseudo-parameter the simulated magnetization curves are somewhat similar to those modelled with the classical Preisach model (the slopes are suppressed to zero). This only partially removes the problem with the JA model, however at this point we can refer to the statements by E. Della Torre et al. as a partial justification for this approach: “In general, the slope of the $M - H$ curve is different before and after a turning point. The reason for the difference in slope is that before the turning point, the change in magnetization is due to both the reversible and the irreversible processes. After the turning point, only the reversible process contributes. This is due to the fact that the irreversible process does not return any energy to the applied field; hence, $\mathrm{d}M/\mathrm{d}H$ is zero for this process” [14].

A clear interpretation of this unusual model behaviour can be found in the paper by Zirka et al. [29]. The presence of negative or zero susceptibility fragments of the magnetization curve is due to the fact that in the fundamental model equation given as an ordinary differential equation (ODE) there is a difference between total magnetization (or its irreversible term, in dependence on model version) and the so-called anhysteretic magnetization, which is supposed to represent the lossless case (the purely reversible process). We believe that separation of total magnetization into reversible and irreversible components is devoid of physical meaning and a description consistent with the laws of irreversible thermodynamics should rather rely on separation of field strengths. Using the analogy of lumped equivalent circuits used commonly in electrical engineering we can envisage the hysteresis operator operating in the “inverse” mode as a black box in which voltage supplies two parallel branches, where one contains a resistor (where the irreversible energy dissipation occurs) and the other one – an inductor (since the electrical angle is equal to 90 degrees then, there is no energy dissipation and this branch represents the reversible process) [30].

What complicates the description even worse, the argument in the aforementioned ODE is the so-called effective field, which contains the so-called Weiss term, which is proportional to total magnetization. Thus, there exists an implicit coupling of irreversible magnetization being a part of total magnetization with the “anhysteretic” magnetization, which should describe just the purely reversible process.

An example of description devoid of the above-described deficiency is the GRUCAD model, proposed by the Brazilian research group in [5]. The application scope for the GRUCAD model is similar to that for the JA formalism – this model also provides an averaged material response. Despite the model developers called their approach “a modified Jiles method”, we believe that this formalism deserves its own name since it has unique features and is consistent with the laws of irreversible thermodynamics. The basic idea of the model is that separate, decoupled sets of equations describe reversible and irreversible magnetization processes. Hysteresis loop, whose area represents energy losses (computed for an elementary volume from the integral $\oint H \mathrm{d}B$) is obtained by introducing offsets (shifts) from a single valued curve crossing the first and the third quadrants of the $H-M$ plane along the H axis and not along the M axis like in the JA model. The input variable is magnetic flux density and the output is total field strength, what makes the model interesting for practitioners. The concept of the GRUCAD model [31−33], as well as of other ones derived from it, may be traced back to the papers by Bergqvist from 1997 and by Chua and Stromsoe from 1971 [34, 35]. Bergqvist has carried out a theoretical study concerning the foundations of thermodynamically consistent hysteresis model in which hysteresis loop is obtained by the introduction of appropriate offsets from the anhysteretic curve (purely reversible curve) along the H axis. The irreversible effects result from contributions of a number of pinning sites. Chua and Stromsoe introduced a general relationship which is applicable to a wide classes of hysteresis curves in the form of the following relationship

$$\frac{\mathrm{d}y}{\mathrm{d}t} = h(y) g \circ \left(x(t) - f(y) \right), \tag{5}$$

where h was interpreted as a weighting function, f and g were interpreted as the restoring (thus reversible) and dissipative (thus irreversible) functions, respectively and the symbol $\circ$ denoted the composition operator.

The GRUCAD model is devoid of the deficiencies of the Jiles-Atherton model, what can be assessed from exemplary modelling results presented in [36]. It can be seen that the modelled First Order Reversal Curves have positive slopes (their differential susceptibilities are greater than zero) also at their origin, Fig. 5.

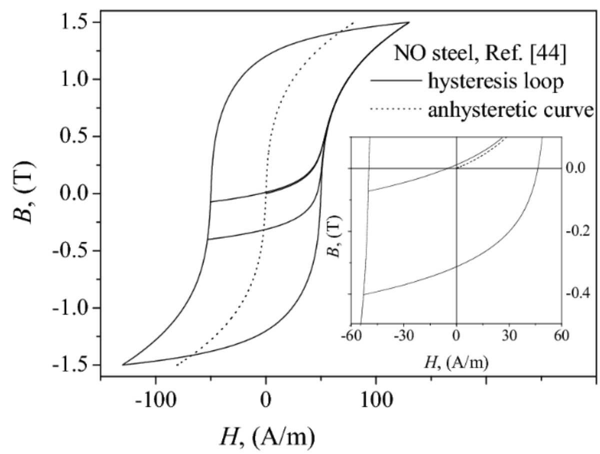

Fig. 5 – *Simulated First Order Reversal Curves for a sample of non-oriented steel* [36].

In the original formulation the GRUCAD model consists of two equations that describe the purely reversible (anhysteretic) process

$$H_{an} = B/\mu_0 - M_s\left(\coth\lambda - 1/\lambda\right), \tag{6}$$

$$\lambda = \frac{1}{a}\left[\left(1-\alpha\right)H_{an} + \alpha B/\mu_0\right] \tag{7}$$

and two subsequent equations that define the irreversible (hysteretic) behavior

$$\frac{dH_h}{dB} = \frac{H_{Hs}\left(\coth\lambda_H - 1/\lambda_H\right) + H_h}{\gamma\delta}, \tag{8}$$

$$\lambda_H = \frac{H_h + \delta H_{Hs}}{a_H}, \tag{9}$$

where $\delta = \pm 1$ is the sign of dB/dt.

The resultant field strength for both branches is determined from summing the irreversible and reversible field strength contributions,

$$H = H_{an} + H_h. \tag{10}$$

Since the anhysteretic curve is unambiguously defined for a closed loop as the middle curve between the ascending and the descending loop branches [37−39] (field strength values are determined from averaging field values determined at the same magnetization/flux density levels, Fig. 6) and the model equations are decoupled, it is possible to carry out estimation in two steps. In the first step the data concerning the measured loop branches is interpolated and the anhysteretic curve is recovered as the middle curve, the parameters α and a are determined; subsequently the measured loop data are corrected to take into account the shearing effect introduced by the anhysteretic curve (this is similar to correcting the magnetic characteristics by the offset introduced by the air gap in an open circuit) and from the second set of equations the remaining the model parameters (H_{Hs}, a_H, γ) are estimated. One of the benefits of such an approach is that the possibility that the estimation procedure gets stuck in a local minimum is avoided, thus more realistic values of model parameters can be estimated.

Fig. 6 depicts in more detail an exemplary major hysteresis loop and two curves which may be recovered from it using averaging along the field strength axis (this is the initial magnetization curve, shown as blue dotted line) and along flux density/magnetization axis (this is the anhysteretic curve). The hysteresis loop was simulated using the $T(x)$ model with an arbitrary choice of reduced coercivity set to $a_0 = 3$ (this value has no apparent meaning, it was chosen just for a better visibility). The concept comes from a sketch in [40].

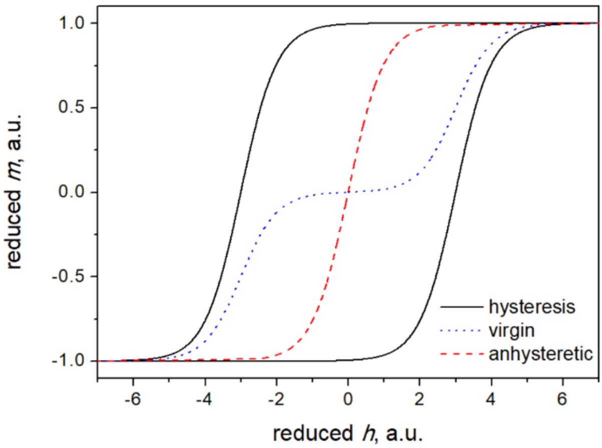

Fig. 6 – *An exemplary hysteresis loop (black solid line) and corresponding virgin (dotted blue line) and anhysteretic (dashed red line) curves.*

The two-step algorithm presented in this section is by no means limited to the GRUCAD description, since the summation of reversible and irreversible field strengths should hold for any hysteresis model. In order to illustrate the approach, we carry out exemplary computations using artificially generated data with the use of the $T(x)$ model, which is simpler in implementation. The results are included in the Appendix.

As far as the $T(x)$ model is concerned, the formalism was developed as a relatively simple general-purpose computational tool, capable of describing a wide range of phenomena that may occur in practice. The model developer used dimensionless units for the quantities x and $y = f(x)$. In Ref. [41] the model was recast into physically meaningful units and the quantity x was interpreted as the reduced "effective field", what could suggest its use as a valuable alternative to the Jiles-Atherton model, devoid of the deficiencies of the JA approach. In the $T(x)$ model there are no problems with negative susceptibilities after field reversals, described in the previous section. The anhysteretic curve connects the loci of loop tips, thus it is equivalent to the so-called commutation curve. It is interesting that the analytically solvable solution to the equation for the inverse anhysteretic curve exists in the $T(x)$ model in which an additional reversible term (usually skipped for simplicity in the model equations) is neglected. In this paper we assume that the application scope for the $T(x)$ model is the same as for the JA and GRUCAD models.

In the $T(x)$ approach the branches of the major loop are given with the expressions (for simplicity we limit ourselves to the discussion of the major loop, the description of symmetric minor loops contains an additional term b, dependent on the excitation level, whose role is to provide the closure of loop tips)

$$y = \tanh\left(x \mp a_0\right) = T\left(x \mp a_0\right), \tag{11}$$

whereas the anhysteretic curve is given with the expression

$$y_{\mathrm{anh}} = 0.5\left[T\left(x + a_0\right) + T\left(x - a_0\right)\right]. \tag{12}$$

In the above-given equations a_0 denotes the reduced coercive field strength, whereas the symbol T stands for hyperbolic tangent.

Availing of the expressions for the sum and the difference of arguments of hyperbolic tangent function one can obtain the expression for the anhysteretic curve in a more compact form [42]

$$y_{\mathrm{anh}} = \frac{T\left(x\right)\left[1 - T^2\left(a_0\right)\right]}{1 - \left[T\left(x\right)T\left(a_0\right)\right]^2}, \tag{13}$$

which may be rewritten as a trivial quadratic equation

$$y_{\mathrm{anh}} T^2\left(a_0\right) X^2 + \left(1 - T^2\left(a_0\right)\right) X - y_{\mathrm{anh}} = 0 \tag{14}$$

and solved immediately for $X = T(x)$ (the positive root is considered in order to avoid complex solutions). Subsequently the corresponding value of x on the anhysteretic curve corresponding to the prescribed value of y_{anh} is computed using inverse hyperbolic tangent function $x = T^{-1}(x)$. Thus it can be stated that for the simplest case an analytical solution to the inverse anhysteretic curve may be found.

3 Discussion

An important conclusion from the analysis of the above-given expression (14) is that in reality the irreversible and reversible processes are mutually coupled for a given hysteresis loop. This concept is similar to the reciprocity theorem in electromagnetism.

One might be curious why the interpretation of x as the "effective field" in the $T(x)$ model does not lead to similar problems as in the JA formalism. The explanation is simple. The only input argument for the (inverse) anhysteretic curve is reduced magnetization (let us recall the statement by Atherton and Schönbächler [26]), on the other hand the "effective field" may be perceived merely as a specific affine transformation [42].

As pointed out already, there are several approaches to take into account reversibility in the $T(x)$ model [13]. However even the simplest method proposed by J. Takács (additional linear term proportional to x) makes the model equations unsolvable in analytical way. However, the equations may be easily solved numerically. Extending the concept advanced by Sablik and Langman [43] who analyzed ferromagnetic samples subject to mechanical stress in a recent publication we suggested that it would be more correct to speak of anhysteretic surface instead of single anhysteretic curve. We also analyzed the effect of DC bias which leads to the occurrence of asymmetric hysteresis curves [44]. However some other phenomena affecting the shapes of hysteresis curves may also be considered in a similar way.

4 Conclusion

In the present paper we tempted to point out the attention of readers to the issue how important it is to include reversibility in several scalar hysteresis models. Using the language of electrical engineering we can state that reversibility may be interpreted as a quantity somewhat analogous to passive power.

As pointed out at the beginning of the paper, we considered here only the reversibility issue for the scalar case, when the vectors $\boldsymbol{H}$, $\boldsymbol{M}$ and $\boldsymbol{B}$ are aligned. The readers are however warned that this is a serious simplification, on the other case it refers to typical conditions for material characterization. In a general case one should work with vectorized versions of the afore-discussed models. However, the vectorization in some cases is itself a non-trivial issue.

5 Appendix – Matlab codes

```
y = (1:-0.01:-1)';
Hc1 = (3 + 0.05*3*randn(1, length(y)))';
% for ascending loop branch
Hc2 = (3 + 0.05*3*randn(1, length(y)))';
%for descending loop branch
% the mean value of coercive field strength assumed to be
% equal to 3
% assumed five percent standard deviation from this value
% this approach may simulate e.g. grain size distribution
% assuming “skewing” of loop branches by 0.01 x
%(reversibility factor, as proposed by Takacs)
x1 = @(y, Hc1, x)  tanh(x-Hc1) +0.01*x – y
% function handle defined for fzero
x2 = @(y, Hc2, x) tanh(x+Hc2) + 0.01*x – y
% function handle defined for fzero
```

```
for i =1:length(y)
 X1(i) = fzero(@(x) x1(y(i), Hc1(i), x), 0);
 X2(i) = fzero(@(x) x2(y(i), Hc2(i), x), 0);
end
% vector [y, X1] stores the loop branch for increasing h
% vector [y, X2] stores the loop branch for decreasing h
```

The resulting loop branches for some arbitrary seed of random number generator are shown in Fig.7.

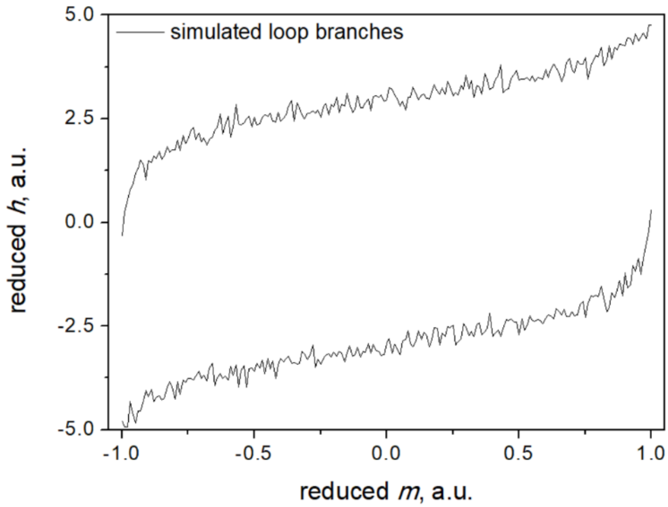

Fig. 7 – *Exemplary simulated loop branches.*

```
Xan = 0.5*(X1+ X2);
```

The reconstructed anhysteretic curve is of course “flickering”; a smoother curve may be obtained using library function movmean, e.g.

```
mXan = movmean(Xan, 3) or movmean(Xan, 5).
```

The second argument of the function denotes the number of data points used.

```
X1cor = X1 – Xan; X2cor = X2 – Xan; mean(X1cor) = 2.9284;
mean(X2cor) = -2.9284
```

The averaged values of coercivity are close to the preset value equal to 3.

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