SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 1, No. 1, November 2003, 111 - 121

Influence of Phase Noise and Interchannel Interference on the Performance of Optical Heterodyne FSK Receiver

M. Stefanović[1], D. Milić

Abstract: The moments approach is generally considered a systematic way to perform the analysis of coherent optical systems. Exact moments of the filtered signal corrupted by phase noise enable the construction of the Gaussian quadrature rule, which may be used to calculate the system error probability. In this paper, we extend the method to a wider class of systems to include the cases where the interchannel interference may be significant. We derive the essential equations in the matrix form and compare the moments approach with numerical simulation and Fokker-Planck approach. To illustrate the results, we apply the moment's method to two-channel optical heterodyne FSK system with dual-filter receiver structure, and evaluate the required channel spacing to have less than 1 dB penalty due to crosstalk.

Keywords: Phase noise, Optical communication, Envelope detection, Interchannel interference, Frequency shift keying.

1 Introduction

Considerable efforts have been devoted to theoretical description of coherent optical systems, in order to account accurately for the influence of laser phase noise on the system performance. During the past decade, several solutions to the problem have been presented in the literature [1-7]. Among the solutions, the most widely used are the results of Taylor expansion method [1] and the moments approach [2, 4, 6]. Both of the methods have been used to describe a variety of modulation/demodulation schemes in the presence of phase noise [8-15], and the Taylor expansion has been used to set up some detailed models of multichannel systems [12, 13].

There have been previous attempts to include the impact of interchannel interference on the FSK system performance, using the moment's approach [9, 11]. However, the potentials of the moments approach were not used to their maximum, and the results are mostly qualitatively valid. In [11], for the sake of simplicity the authors neglect the phase shift between the consecutive samples and use only the rough worst-case approximation. This has lead to only an approximate model, even with no influence of phase noise. On the other hand, a model corresponding to the mean values with uni-

[1] Faculty of Electronic Engineering of Niš, Department of Telecommunications, Beogradska 14, 18000 Niš Serbia and Montenegro, Phone: +381 18 529 309, Fax: +381 18 524 931, E-mail: misa@elfak.ni.ac.yu

formly distributed interference phase is considered in [9]. In both cases [9, 11], the effects of time shift between the interfering channels were neglected, and only the systems with ideally synchronized channels were considered. A comprehensive worst-case analysis of ASK systems, which includes the aforementioned effects is outlined in [12]. However, the method proposed in [12] uses the leading order Taylor expansion to account for the phase noise influence, together with an approach based on the inverse Fourier transform to compute the bit-error rate, as opposed to conditional error probability approach [10, 11] that is generally more accurate. In this paper, we outline a procedure that combines the good sides of both the approaches -the worst-case analysis of [12], and the conditional error probability approach [10]- to yield the results that should be in closer agreement with the real system performance. We apply the procedure to heterodyne dual-filter FSK receiver, and calculate the required channel separation for a two-channel system.

2 Moments Evaluation

Analysis of coherent optical system performance, including the effects of interchannel crosstalk, requires the knowledge of probability density function (pdf) of the following random variabl

$$\frac{1}{\tau}\int_0^\tau \left(e^{j\varphi(t)} + e^{j(\theta+2\pi d_{el}t+\phi(t))}\right)dt\ , \tag{1}$$

or - in envelope detection schemes- its squared modulus. Random phase processes $\varphi(t)$ and $\phi(t)$ are considered independent [9] Brownian motions [1] with diffusion constants $2\pi\Delta\nu$, d_{el} is channel separation and θ is interference offset phase - constant over the one bit duration. Using Taylor expansion of the interference, the leading asymptotic behaviour is obtained as [12]:

$$\frac{1}{\tau}\int_0^\tau e^{j\varphi(t)}dt + je^{j\theta}\,\frac{1-e^{j2\pi d_{el}\tau}}{2\pi d_{el}\tau} + e^{j\theta}\,\frac{\phi(\tau)e^{j2\pi d_{el}\tau}-\phi(0)}{2\pi d_{el}\tau} \tag{2}$$

In the above equation, it is convenient to identify the interference as a Gaussian random variable with mean r, and variance $\frac{\Delta\nu\tau}{2\pi(d_{el}\tau)^2}$. Therefore, it is possible to include the last term of the previous equation with other Gaussian noise contributions, such as shot and receiver noises [9-11]. However, the deterministic interference term r is more complicated to account for.

Let the moments $\hat{\mu}_{m,n}$ of X be defined as:

$$\hat{\mu}_{m,n} = E\left[X^m \overline{X}^n\right]. \tag{3}$$

where the overbar stands for complex conjugation. Exact moments $\mu_{i,j}$, of the filtered phase-noisy signal without the influence of interference, are known in symbolic form [2, 4, 6], and they can be used to express $\hat{\mu}_{m,n}$, as we will show.

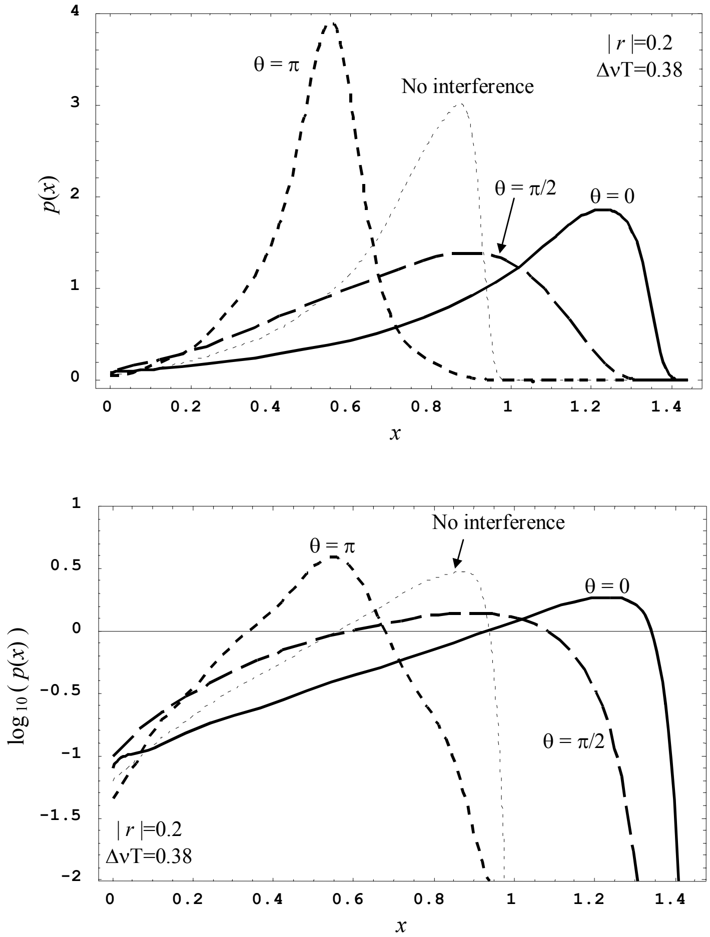

Fig. 1 - *Probability density functions of the envelope detector output, with and without deterministic interference. Curves are reconstructed from the first 12 moments using maximum entropy approach.*

Introducing the notation $X = r + z$, where

$$r = e^{j(\theta+\pi d_{el}\tau)} \frac{\sin(\pi d_{el}\tau)}{\pi d_{el}\tau}, \tag{4}$$

$$z = \frac{1}{\tau}\int_0^\tau e^{j\varphi(t)}dt \; , \tag{5}$$

according to (2), the moments $\hat{\mu}_{m,n}$ are written as

$$\begin{aligned}\hat{\mu}_{m,n} &= E\left\{(r+z)^m(\bar{r}+\bar{z})^n\right\} \\ &= E\left\{\sum_{i=0}^{m}\sum_{j=0}^{n}\binom{m}{i}\binom{n}{j} z^i \, \bar{z}^{\,j} \, r^{m-i} \, \bar{r}^{\,n-i}\right\} \\ &= \sum_{i=0}^{m}\sum_{j=0}^{n}\binom{m}{i}\binom{n}{j}\mu_{i,j} \, r^{m-i} \, \bar{r}^{\,n-i} \quad .\end{aligned} \tag{6}$$

It is convenient to write the forward equation in the matrix form:

$$\hat{\mu}_{m,n} = \langle\bar{\rho}\rangle_m \langle M\rangle_{n,m}^T \langle\rho\rangle_n^T \, , \tag{7}$$

where $\langle M\rangle_{n,m}$ denotes the moment-matrix of the random variable z, namely:

$$\langle M\rangle_{n,m} = [\![\, \mu_{i,j} \,]\!]_{\substack{i=1,2,\dots,n \\ j=1,2,\dots,m}} \; . \tag{8}$$

The row-vector $\langle\rho\rangle_k$ is defined as $\langle\rho\rangle_k = [\, r_i \,]_{i=1,2,\dots,k}$, where r_i are given by:

$$r_i = \binom{k-1}{i-1} r^{k-i} \; . \tag{9}$$

The moments $\hat{\mu}_{k,k}$, which are the moments of the random variable $|X|^2$, may therefore be obtained as:

$$\hat{\mu}_{k,k} = \langle\bar{\rho}\rangle_k \langle M\rangle_{k,k}^T \langle\rho\rangle_k^T \, . \tag{11}$$

Of course, due to symmetry of the moments $\mu_{i,j}$, the moment-matrix is also symmetric: $\langle M\rangle^T = \langle M\rangle$.

The described procedure may be generalized to yield the following result:

$$\left\langle \hat{M} \right\rangle_{k,k} = \left\langle \bar{R} \right\rangle_{k,k} \langle M\rangle_{k,k}^T \langle R\rangle_{k,k}^T \, , \tag{12}$$

where the matrix $\langle R\rangle_{k,k}$ is defined as

$$\langle R\rangle_{k,k} = [\![\, r_{i,j} \,]\!]_{i,j=1,2,\dots,k} \tag{13}$$

and the $r_{i,j}$ are given by

$$r_{i,j} = \begin{cases} \binom{j-1}{i-1} r^{j-i} & , \quad i \le j \\ 0 & , \quad i > j \,. \end{cases} \tag{14}$$

The moment-matrix $\langle \hat{M} \rangle$ of the random variable X may not be real, as this is obvious from (4) and (6). However, the moments on the main diagonal, which represent the moments of the random variable $|X|^2$, are all real. The matrix also satisfies: $\langle \hat{M} \rangle^* = \langle \hat{M} \rangle$, and thus is Hermitian [18].

The range of values that random variable $|X|^2$ takes should also be considered. The variable is represented as a square modulus of a sum of two vectors:

$$|r + z|^2 = |r|^2 + |z|^2 + 2|r||z| \cos \Delta\theta \,, \tag{15}$$

where $\Delta\theta$ is a random phase difference between the deterministic interference r and the random variable z. Since the variable z satisfies $0 \le |z|^2 \le 1$, and it is obviously $-1 \le \cos \Delta\theta \le 1$, it is possible to bound the forward expression. It turns out that the general inequality is $x_L \le |X|^2 \le x_U$, where the lower and upper bounds are:

$$x_L = \begin{cases} 0 & , \quad |r| \le 1 \\ (|r| - 1)^2 \, , & |r| > 1 \end{cases} , \qquad x_U = (|r| + 1)^2 \,. \tag{16}$$

The result may be useful in numerical reconstruction of the pdf from its moments using, for example, the maximum entropy estimation.

3 Application to FSK System

The moments (11) may be used in a worst-case performance analysis, as we will show on the FSK system example. We consider a receiver model as shown in Fig. 3. It is a heterodyne polarization control receiver with dual-filter structure. Frequency deviation of the incoming FSK signal is considered large and the correlation effects between the two receiver branches are neglected [8]. IF filtering is performed using equivalent integrate-and-dump filters with central frequencies tuned to the FSK signal frequencies, and with integration time τ. Postdetection filter is a summation device that averages Md consecutive detected samples during the bit interval. Shot noise is considered the dominant Gaussian noise factor; other Gaussian noise contributions can also be easily included in the analysis. Under these conditions, the error probability is computed as derived in [10, 15].

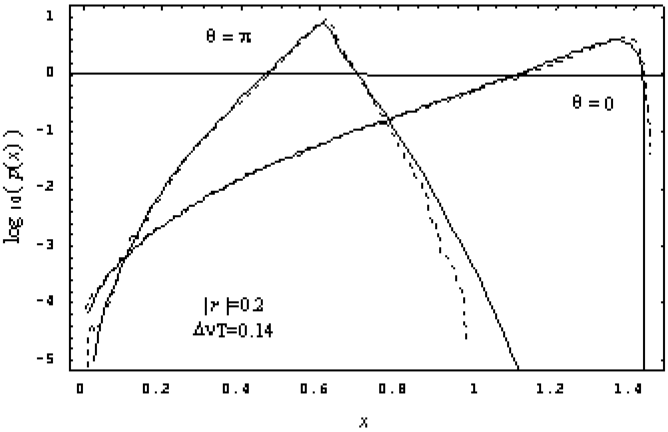

Fig. 2 - *Pdf of the envelope detector output with "best" and "worst" case interference, in logarithmic scale. Curves: full - Fokker-Planck approach, dashed - numerical simulation of the random variable X.*

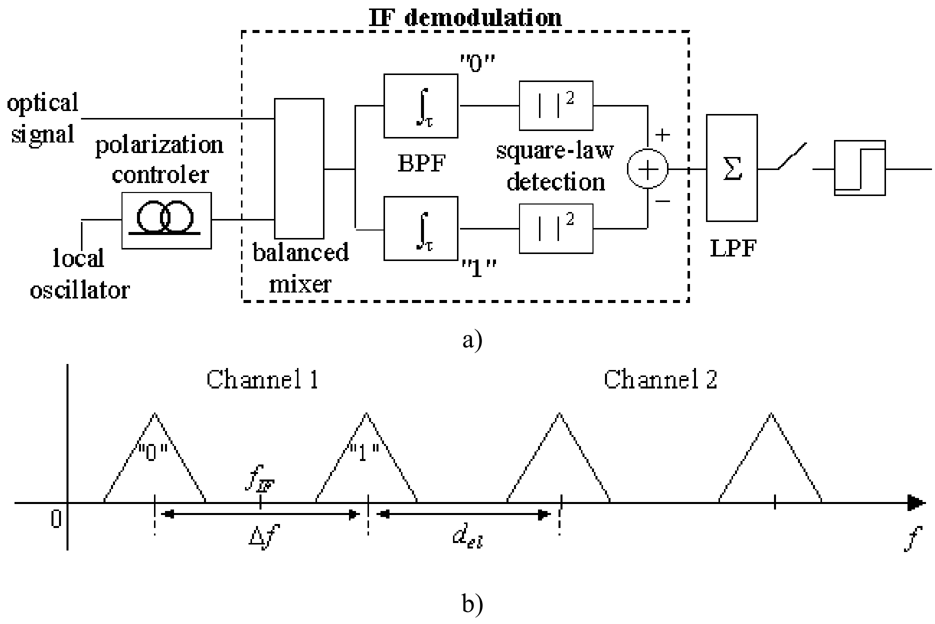

Fig. 3 - *(a) Block diagram of the FSK receiver model and (b) The schematic of channels after balanced detection.*

To include the effects of interchannel interference, we use the results of [11] with some modifications. We consider a two-channel heterodyne model with low intermediate frequency and ideal envelope detection (see e.g. [16]). Interchannel interference is therefore the crosstalk from the other channel, which is separated in the electrical domain by the spacing d_{el}. The crosstalk will have different influence on the transmission of binary "0" and "1". When the binary "0" is transmitted, the crosstalk can never be constructive since its squared modulus in "1" branch impairs the decision variable; hence the interference phase is irrelevant. The amount of crosstalk changes with channel spacing and with time shift between the data, as explained in [12]. During the transmission of binary "1", the effect of crosstalk additionally depends on the relative signal phase in the interfering channel. Depending on the interference phase, the crosstalk can be either constructive or destructive (see Fig. 1). Since the frequency deviation Δf is considered large, the interference on the "0" branch may be neglected.

For the given system model, the worst-case error probability is computed as:

$$P_e = \frac{1}{2}P(0/1) + \frac{1}{2}P(1/0), \tag{17}$$

where $P(0/1)$ is the worst-case probability of detecting "0" when binary "1" is transmitted, and vice-versa for $P(1/0)$. The probabilities are computed based on the results of [11], which have been modified to reflect the differences in system models and to include the more accurate statistics of phase-noisy signal with crosstalk. Worst-cases are then found by numerical search over the crosstalk time shift and initial phase.

The following steps outline the procedure of performance evaluation of the FSK receiver:

1) Compute the error probability for the single-channel system as in [10]. Optimise the integration time and the number of samples to obtain the best performance for the given total laser line width.

2) With Md optimised in the previous step, and for the given channel spacing, compute $P(1/0)$ for the worst-case time shift $\tau_2=1/(2d_{el})$, (see [12]), during the last sample.

3) Compute $P(0/1)$ for arbitrary time shift, initial phase and transition sample, taking into account the interference phase shift over each sample [12]. For this purpose, use (11) and Appendix A to compute the appropriate moments. Using the computer search over the variables, find the worst-case performance.

4) Using (17) and two previous steps, find the sensitivity penalty relative to the ideal single-channel case with no phase noise.

Concerning the step 3, it is not straightforward to obtain the moments of the decision variable with arbitrary interference phase shift during each sample. To this end, a procedure that enables the computation is derived in Appendix A. Once the appropriate moments are calculated, a Gaussian quadrature rule can be constructed in order to compute the performance [10, 11]. The procedure is also applicable to step 2, with simpler conditions that there is no interference in the signal branch, i.e. all W_i (see Appendix A) are equal.

4 Numerical Results and Discussion

In Fig.1, we compare the pdf's at the square-law envelope detector output, with, as well as without interference. The curves are reconstructed using maximum entropy method and first 12 moments. We have also obtained the densities using the Fokker-Planck equation [1, 2, 5] to find the joint density of real and imaginary parts of z, and then numerically computed the densities of $|X|^2$ for the same r-values. The results showed very good agreement with those shown in Fig.1. In Fig.2 we show the pdf resulting form Fokker-Planck approach compared to the results of numerical simulation of variable X. The agreement is apparent and general behaviour of the pdf curves is also similar to Fig.1.

Fig.4 shows sensitivity penalty of the FSK receiver due to phase noise and interchannel crosstalk. Penalties are calculated relatively to the sensitivity of the single channel receiver with no phase noise influence, which is 40 photons per bit. In the limit of no phase noise, the two-channel system considered requires channel separation d_{el} of 2.7 times the bit rate in order to operate within 1 dB penalty. Phase noise generated by the lasers with total line width of 8% of the bit rate causes further sensitivity degradation. Wider bandwidths are required to contain the signals and the best single-channel sensitivity is obtained for Md=2, resulting in about 0.6 dB penalty without any crosstalk. Additional penalty due to crosstalk from the second channel will be less than 1 dB when the channel separation is above 3.6 times the bit rate. However, if the two channels are operated with exactly the same bit rate, and are synchronized, this should allow closer channel separation of about 2 times the bit rate. The situation is also beneficial for the system without phase noise, where the channel spacing equal to the bit rate would suffice (not shown in Fig.4).

When the total laser line widths equal 26% of the bit rate, the optimum Md value is 3 and the required channel spacing is about 5.5 times the bit rate. In this case, the synchronization of the channels cannot reduce the required channel separation, although somewhat smaller penalties are expected. For the line widths equal to the bit rate, optimum Md is 7 and the required channel separation rises to about 12, while the effects of synchronization are less noticeable. Therefore, synchronization may enable closer channel separation only when the laser line widths are relatively small. When the line widths are close or even larger than the bit rate, the difference between synchronized and non-synchronized systems becomes negligible.

We would like to emphasize that the performance can be computed asymptotically accurate as long as local laser and neighbouring channel transmitting laser have negligible line widths with respect to the transmitting laser. Moreover, if the neighbouring channel transmitter line width is non-negligible, but nevertheless small, the leading order asymptotic description (2) of interference term is expected to be valid. As a whole, this is a reasonable degree of accuracy, somewhat better than the previous results. However, in the real system, all the lasers are expected to have same line widths, and the FSK results of this paper should therefore be considered approximate.

V. Conclusion

In this paper, we have presented a procedure that enables the use of moments approach in detailed analysis of coherent optical systems impaired by phase noise and interchannel interference. Probability density functions obtained with maximum entropy reconstruction from the moments have shown close agreement with numerical simulation and the Fokker-Planck approach, indicating a good accuracy of the model. Furthermore, we have set up a model of a heterodyne FSK receiver and applied the procedure to performance evaluation of the two-channel system. We have found that the required channel spacing for 1 dB crosstalk penalty is about 2.7 times the bit rate in the worst-case situation without any influence of phase noise. When total laser line width equals the bit rate, the required channel spacing rises to at least 12 times the bit rate. Somewhat closer channel spacing may be achieved by synchronizing the two channels, but the operation is expected to yield significant results only if the laser line widths are relatively small.

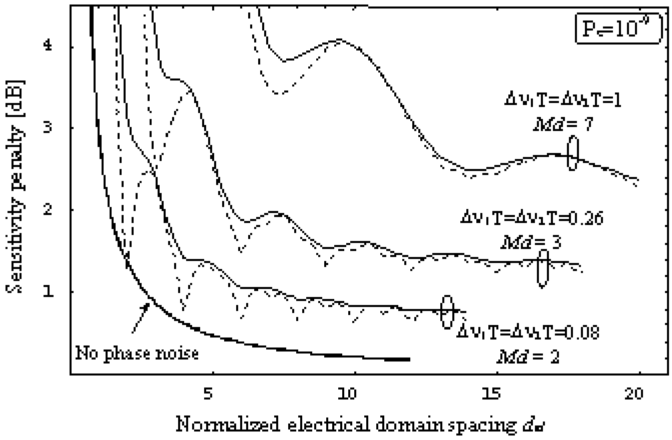

Fig. 4 - *Worst-case sensitivity penalty of FSK receiver, versus the electrical domain channel spacing. Curves: full - non-synchronized channels; dashed - synchronized channels.*

Appendix
Moments of the sum of independent variables $|X_i|^2$

Define $\xi_m = \sum_{i=1}^{m} |X_i|^2$ and $\eta_{m+1} = |X_{m+1}|^2$. Moments of the sum of m+1 independent variables $|X_i|^2$ may be obtained by the following recurrence approach:

$$\zeta_n^{\Sigma(m)} = E\left\{\left(\sum_{i=1}^{m} |X_i|^2\right)^n\right\} = E\left\{\xi_m^n\right\} \text{ and} \tag{A1}$$

$$\begin{aligned}\zeta_n^{\Sigma(m+1)} &= E\left\{\left(\sum_{i=1}^{m} |X_i|^2 + |X_{m+1}|^2\right)^n\right\} = E\left\{(\xi_m + \eta_{m+1})^n\right\} \\ &= E\left\{\sum_{k=0}^{n}\binom{n}{k}\xi_m^{n-k}\eta_{m+1}^k\right\} = \sum_{k=0}^{n}\binom{n}{k}\zeta_{n-k}^{\Sigma(m)}\hat{\mu}_k^{(m+1)}\end{aligned} \tag{A2}$$

Again, the matrix formulation is convenient because the recursion process can be replaced by the multiplication of m matrices. The matrix equation is expressed as

$$\begin{aligned}\left\langle \zeta^{\Sigma(m+1)} \right\rangle_k &= \left\langle \zeta^{\Sigma(m)} \right\rangle_k \cdot \left\langle W_{m+1} \right\rangle_{k,k} \\ &= \left\langle \zeta^{\Sigma(m-1)} \right\rangle_k \cdot \left\langle W_m \right\rangle_{k,k} \cdot \left\langle W_{m+1} \right\rangle_{k,k} \\ &= \ldots = \left\langle \mu^{(1)} \right\rangle_k \cdot \prod_{i=2}^{m+1} \left\langle W_i \right\rangle_{k,k} \quad ,\end{aligned} \tag{A3}$$

where $\left\langle \zeta^{\Sigma(m)} \right\rangle_k$ denotes the moment row-vector of the sum of m variables, defined as

$$\left\langle \zeta^{\Sigma(m)} \right\rangle_k = \left[\zeta_{i-1}^{\Sigma(m)} \right]_{i=1,2,\ldots,k} . \tag{A4}$$

$\left\langle \hat{\mu}^{(i)} \right\rangle_k$ is the moment row-vector of a single variable $|X_i|^2$, and the matrix $\left\langle W_n \right\rangle_{k,k}$ is defined as

$$\left\langle W_n \right\rangle_{k,k} = \left[\!\left[w_{i,j}(n) \right]\!\right]_{i,j=1,2,\ldots,k} , \tag{A5}$$

with elements

$$w_{i,j}(n) = \begin{cases} \binom{j-1}{j-i} \hat{\mu}_{j-i}^{(n)} & , \quad i \le j \\ 0 & , \quad i > j \end{cases} . \tag{A6}$$

By proceeding one step further from (A3), it is easy to identify that the moment vector $\left\langle \zeta^{\Sigma(m)} \right\rangle$ equals the first row of the matrix $\prod_{i=1}^{m} \left\langle W_i \right\rangle$.

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