

## On the Sensitivity of the Recursive Filter with Arbitrary Order Predictor in DPCM System

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**Abstract:** Sensitivity analysis of the recursive filter in a DPCM system, with arbitrary order predictor, is presented in this paper. Relations for sensitivity, related to predictor coefficients, are derived both in cases of stable and unstable systems. In this way, we presented new information which can improve design of the system in the sense of better performances via parameters (coefficient) adjustment. Given relations were verified for the case of real system in three different configurations: with the second, third and fifth order predictors.

**Keywords:** Parametric sensitivity, Transfer function, Differential pulse code modulation, Predictor coefficients.

### 1 Introduction

Sensitivity analysis determines the influence of change of parameters or disturbances on the systems state coordinates [1 – 3]. In this paper, we will consider only parametric sensitivity. Parametric sensitivity is of great importance during design of systems [4 – 6], because we always need to know how changes of some system parameters effect on system performances as a whole. Sensitivity of telecommunications systems was considered in [7 – 9], both for continuous and discrete systems. In this paper, sensitivity analysis is performed for one real discrete telecommunication system.

DPCM transmission system is a nonlinear feedback system [10]. Because of negative closed loop, although basically a telecommunication system, DPCM is also suitable for analysis in terms of the control systems theory: stability [11], sensitivity, etc. Predictor is one of the most important elements of each DPCM telecommunication system. Generally, it is nonlinear element, but it can be approximately considered as linear in practice, as given in this paper.

Parametric sensitivity of a recursive filter with arbitrary order predictor is given in the paper. Sensitivity values were obtained related to the predictor coefficients. Because predictor coefficients values have direct impact on system

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performances, defining these values is very important step. Better adjustment of these values causes less deviation between real and predicted value of the signal. This is the reason why it is important to know sensitivities related to the each coefficient and on which coefficient (parameter) the system is the most sensitive.

The paper is organized as follows. Section 2 describes the DPCM transmission system. Mathematical relations and theoretical conclusions for parametric sensitivity of the recursive filter are given in Section 3. Derived relations are verified in three different cases in Section 4.

## 2 DPCM Transmission System

Differential pulse code modulation (DPCM) is a technique of converting an analog signal into a digital in which analog signal is sampled and then the difference between the actual sample value and its predicted value is quantized and, finally, encoded by a digital value [10, 12]. The predicted value of the actual sample is based on previous sample or samples.

The block diagram of a DPCM system is given in Fig. 1. The encoder (Fig. 1a) consists of the quantizer, inverse quantizer and predictor. Linear recursive filter in the feedback loop is denoted with  $R$ . Also, in Fig. 1a, the additional subsystem for the adaptive prediction is shown (buffer and estimator of predictor coefficients connected with dotted lines), forming an ADPCM encoder. Input signal is sampled with equal time period,  $T_0$ . In this way, we obtain signal samples from  $N$  previous moments ( $x_i, i=1,2,\dots,N$ ). In DPCM system, the difference between the current sample  $x_n$  and its predicted value  $\hat{x}_n$  is very important:

$$d_n = x_n - \hat{x}_n. \quad (1)$$

This difference is quantized and transmitted, whereby quantization error  $e_n$  occurs. This error is superimposed to the signal sample and result is reconstructed signal:

$$y_n = d_n + e_n + \hat{x}_n = x_n + e_n. \quad (2)$$

This is also input signal of linear predictor, the element of great importance for the further analysis.

Generally,  $N$ -th order predictor consists of  $N$  parallel branches. Each branch consists of one delay line and one valuation circuit ( $a_1, a_2, a_3, \dots, a_N$ ). The delay line with the valuation circuit  $a_1$  holds sample of input signal for one sample period  $T_0$ ; the delay line with  $a_2$  holds it for two periods  $2T_0$ , and finally, the last line holds signal sample for  $NT_0$ . After valuating, samples from  $N$  previous

sample periods appear simultaneously at predictor output and form linear prediction of the sample:

$$\hat{x}_n = \sum_{i=1}^N a_i y_{n-i}, \quad (3)$$

where  $a_i, i=1,2,\dots,N$  are predictor coefficients. These coefficients have great influence on quality of signal transmission, i.e. on prediction error. They are defined in advance, according to the class of considered signals [10, 12].

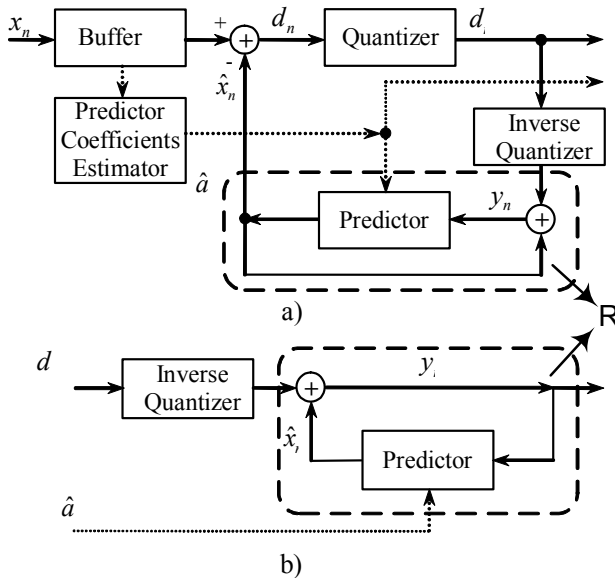


Fig. 1 – DPCM/ADPCM system: (a) Encoder; (b) Decoder.

### 3 Sensitivity of the Recursive Filter with $i$ -th Order Predictor

The main goal of this paper is to present relations for the sensitivity of the linear recursive filter  $R$  with arbitrary order predictor. Therefore, we consider sensitivity of recursive filter with  $i$ -th order predictor. Relation (3) which describes predictor, in  $z$ -domain has the following form:

$$\hat{X}(z) = \left( \sum_{i=1}^N a_i z^{-i} \right) Y(z). \quad (4)$$

Transfer function of  $i$ -th order predictor:

$$W_p(z) = \sum_{i=1}^N a_i z^{-i} = a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \dots + a_N z^{-N}. \quad (5)$$

Transfer function of recursive filter,  $R$ :

$$W_R(z) = \frac{W_P(z)}{1 - W_P(z)} = \left(1 - \sum_{i=1}^N a_i z^{-i}\right)^{-1} = \frac{1}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - \dots - a_N z^{-N}}. \quad (6)$$

Let denote input signal into the recursive filter with  $e_n^*$  (Fig. 1), i.e.  $e^*(z) = d(z) + e(z)$ . Then, predicted value  $\hat{X}(z)$ , which is in the same time the output of the recursive filter, has the following form:

$$\hat{X}(z) = W_R(z) e^*(z) = \frac{1}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - \dots - a_N z^{-N}} e^*(z). \quad (7)$$

In general case, parametric sensitivity for discrete system [4, 7, 8] related to certain parameter  $a_i$  is defined as:

$$u_{a_i}(z) = \frac{\partial y(z, a_0, a_1, \dots, a_N)}{\partial a_i}, \quad i = 0, 1, \dots, N. \quad (8)$$

Using (8) we can determine the sensitivity of arbitrary order recursive filter in DPCM system related to the certain predictor parameter (coefficient):

$$\begin{aligned} u_{a_1}(z) &= \frac{-z^{-1}}{\left(1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - \dots - a_N z^{-N}\right)^2} e^*(z), \\ u_{a_2}(z) &= \frac{-z^{-2}}{\left(1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - \dots - a_N z^{-N}\right)^2} e^*(z), \\ u_{a_3}(z) &= \frac{-z^{-3}}{\left(1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - \dots - a_N z^{-N}\right)^2} e^*(z), \end{aligned} \quad (9)$$

i.e. for  $i$ -th predictor coefficient, we have:

$$u_{a_i}(z) = \frac{-z^{-i}}{\left(1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - \dots - a_N z^{-N}\right)^2} e^*(z), \quad (10)$$

$$\left|u_{a_i}\right|(z) = \frac{|z|^{-i}}{\left|1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - \dots - a_N z^{-N}\right|^2} e^*(z), \quad i = 1, 2, \dots, N. \quad (11)$$

In the case that recursive filter is stable, i.e.  $|z| \leq 1$  ( $|z^{-i}| \geq 1$ ), the following inequality is valid:

$$\left|u_{a_1}\right| \leq \left|u_{a_2}\right| \leq \dots \leq \left|u_{a_N}\right|. \quad (12)$$

Hence, when the recursive filter is in the stability region, sensitivity is increasing with increase of index of predictor coefficient. The system is the most sensitive related to the coefficient with the highest index, and the least sensitive related to coefficient  $a_1$ .

When the recursive filter is unstable, conclusions regarding sensitivity are the opposite. Now, the system is the most sensitive for coefficient  $a_1$ , and the least sensitive for coefficient  $a_N$ , where  $N$  is the predictor order.

#### 4 Case Study

We consider DPCM system with the second, third and fifth order predictor. Recorder speech signal, i.e. low-pass filtered speech signal of 10200 samples (duration 1275 ms) in frequency range of 65-3400 Hz is analyzed.

We record sensitivity functions for each predictor coefficient considering the case when the recursive filter is stable: for:  $\sigma = -0.6\text{kHz}$ ,  $\omega \in [0\text{kHz}-10\text{kHz}]$ ,  $T = 10^{-3}\text{s}$ . Parametric sensitivity should be increasing with increase of the index of predictor coefficient according to (12), inside stability region ( $|z| = e^{|\sigma|T} e^{(\sigma+j\omega)T} \leq 1$ ).

Predictor coefficients are defined in advance, adapted for one tenth of adaption interval ( $M = 20\text{ms}$ ). For the second order predictor, predictor coefficients are:  $a_1 = 1.42$ ,  $a_2 = -0.54$ ; for the third order:  $a_1 = 1.58$ ,  $a_2 = -0.96$  and  $a_3 = 0.30$ .

Obtained sensitivities are shown in Figs. 2 and 3.

Sensitivity related to the coefficient  $a_1$  is the lowest, higher for  $a_2$  (Figs. 2 and 3) and highest for  $a_3$  (Fig. 3) according to the relations derived above.

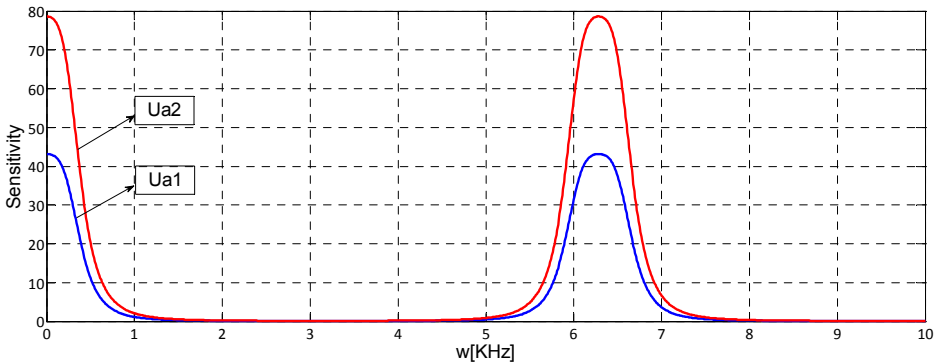


Fig. 2 – Sensitivities for the second order predictor coefficients.

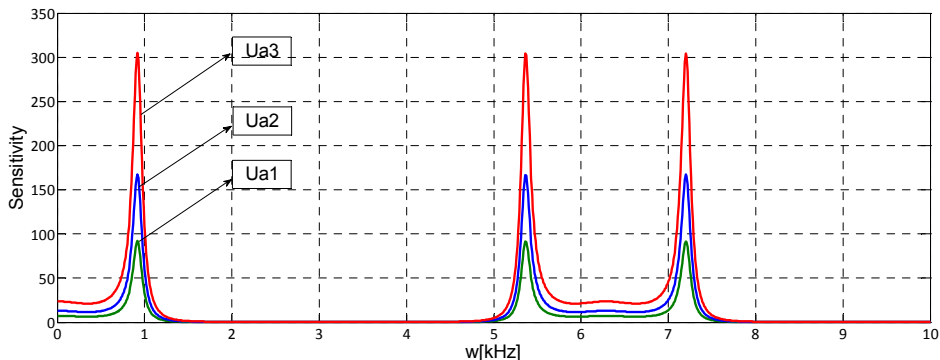


Fig. 3 – Sensitivities for the third order predictor coefficients.

Prediction gain significantly increases up to the second or eventually the third order, then gets into saturation [12]. For the purpose of validation of given relations, we also considered recursive filter with higher order predictor (the fifth order). Predictor coefficients  $(a_1, a_2, a_3, a_4, a_5)$  have the following values: 1.65, -1.19, 0.69, -0.38, 0.14, respectively. It means that relation for predictor in  $z$ - domain (4) becomes:

$$\hat{X}(z) = (1.65z^{-1} - 1.19z^{-2} + 0.69z^{-3} - 0.38z^{-4} + 0.14z^{-5})Y(z).$$

Obtained sensitivities are shown in Fig. 4.

It should be noted that due to the better transparency, symbol for predictor coefficients sensitivity  $U_{a_i}$  is introduced:

$$U_{a_i} = \frac{u_{a_i}(z)}{e^*(z)}.$$

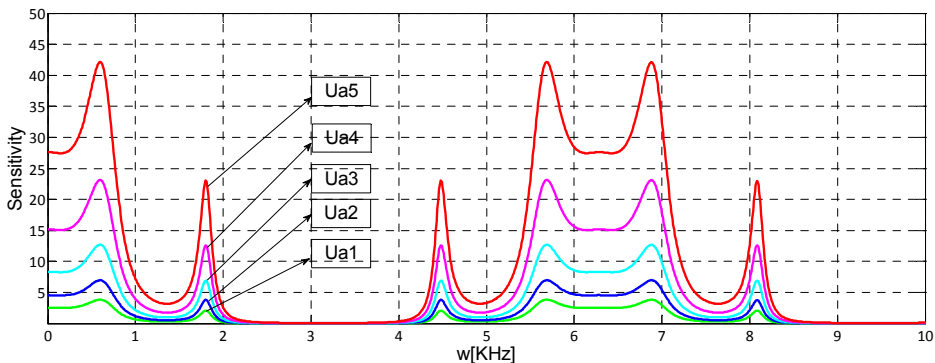


Fig. 4 – Sensitivities for the fifth order predictor coefficients.

As we can see in Fig. 4, sensitivity related to coefficient  $a_1$  is the lowest, higher is for  $a_2$ , and finally the system is the most sensitive for  $a_5$ , in the whole frequency range. In this way, we verified mathematically derived relation (12) in practice.

## 5 Conclusion

One of the most common transmission system, DPCM system and especially its most important part, predictor, is the subject of research in this paper. As the predictor coefficients have great impact on the system performances, it is of main importance to know sensitivities on predictor values changes. For this purpose, general relations for parametric sensitivity for specific predictor coefficient of arbitrary order predictor are given.

Through the mathematical analysis and experiments, for concrete signal and values of predictor coefficients, we reached the following conclusion: stable recursive filter is the least sensitive related to coefficient  $a_1$ , and the most sensitive related to the coefficient with the highest index. We verified derived relations for the case of the second, the third and the fifth order predictor.

Further sensitivity study will possibly contain the whole DPCM system (with quantizer as nonlinear element).

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