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Two Approaches to Solving the Problem of Smoothing Digital Signals Based on a Combined Criterion

Evgenii Semenishchev[1], Igor Shraifel[1], Ilya Svirin[2]

Abstract: The paper presents a method for smoothing signals represented by a single realization of a finite-length random process, under conditions of a limited amount of a priori information about the signal function and statistical characteristics the noise component. The recommendations on the use of parameters affecting the processing speed and the efficiency of smoothing are given. Two solutions are presented to obtain the result of smoothing the signals. The efficiency results are shown for the processing of digital signals. Examples of comparison of simple methods and suggested ones are given.

Keywords: Signal processing, Denoising, Two-criteria method, Smoothing.

1 Introductions

In modern automatic control systems for data collection, processing and transmission, intelligent sensors play a special role. These devices allow for continuous monitoring and transmission of information to a remote terminal. The process of signal conversion is associated with the effect on the measured signal of a random component. To reduce noise, processing is performed immediately after the analog interface of the sensor, and data transfer to subsequent monitoring systems is carried out digitally [1]. Modern sensor systems produce ADC conversion of the received signals and a reduction in the influence of the noise component. In this connection, high demands are placed on the sensitive element and the pre-treatment block. There are technological limitations in the manufacture of the measuring element and the corresponding devices of the analog interface. As a consequence, algorithms for preliminary processing of digital signals are of special interest for increasing the reliability.

The use of digital signal processing methods has found wide application [2]: in automation and control systems, when creating sensors with the possibility of automatic adjustment in the event of possible aging of the sensing element or changes in environmental parameters; in modern antenna systems, in

[1]Don State Technical University, Gagarina sq. 1, Rostov-on-Don, 344000, Russia;
E-mails: sea.sea@mail.ru; shraifel17@mail.ru
[2]CJSC Nordavind, Warsaw highway 125, Moscow, Russia; E-mail: i.svirin@nordavind.ru

the study of atmospheric, hydro- and lithospheric structures, as well as object detection systems; when investigating biomechanical parameters, biometric data collection systems located directly on the object under study; in modern systems of automatic processing of two-dimensional signals obtained from CCD the photo and video cameras, as well as computer vision systems, to reduce the noise effect of video sensors associated with the operation of the communication channel or the defect of the scanning device; in economics and sociology in the study of trends; in information-measuring systems; in computer technology to increase the accuracy associated with the possibility of reducing the interference caused by noise conversion of the signal from analog to digital.

In the general case, the analysis of signals is hampered by the presence of noise having a random character with priori unknown statistical characteristics. Information about the useful component of the signal is also limited. The use in the automation and control systems of the methods of considered in the works of leading scientists, such as V.I. Tikhonov, A.I. Orlov, B.R. Levin, L. Rabiner, B. Golden, etc., is possible only if there is a sufficient amount of a priori information, otherwise, their effectiveness is significantly reduced [3–6].

As a rule, in these cases, the methods of processing are based on minimizing the standard deviation criterion or maximizing the signal-to-noise ratio. The choice of the criterion is due to the amount of a priori information about the problem being solved. In the conditions of a limited amount of information on the useful signal function and the statistical characteristics of noise, the problem is sharply complicated. The presence in the useful component of the points of discontinuity of the first kind imposes additional requirements on the methods of processing.

In this connection, the actual task is to develop methods and algorithms for smoothing the digital signals of measuring complexes and automatic control systems, as well as processing devices simultaneously by several criteria in conditions of a limited amount of a priori information about the function of the useful and noise components.

In modern automatic systems for the collection, processing and transmission of measurement on the signal is affected by various interference. The process of converting a signal into a digital form also introduces a random component into the signal. The main task in the processing of signals is the separation of the useful and the noise component. At the same time, in practice, the criterion of minimum mean square error or the criterion of mean-absolute deviation is used. Each of the criteria has merits and limitations of use depending on the task and a priori information about the components of the input signal [7]. Actual is the task of processing a digital signal based on the objective function of the combined criteria. Of particular interest is the use of multicriterial methods for processing digital signals. When the signal is represented by a single implementation with a limited amount of a priori information about the useful component and statistical information about noise characteristics.

2 Mathematical Model of Signals

Let the input signal be a discrete sequence of values of the measured physical quantity $Y(t_k)$ obtained at equidistant times $t_k = kT$, when $k = \overline{1,n}$ ($T > 0$ – constant). This signal can be considered as an implementation of the random process $Y(t_\text{k})$, which is an additive mixture of useful and noise components. A simplified mathematical model of the input signal is represented in the form [7]:

$$Y(t_k) = s(t_k) + \eta(t_k)\,,\ k = \overline{1,n}\,. \tag{1}$$

when $s(t_k)$ – useful signal component; $\eta(t_k)$ – additive noise component, n – sample size.

The functional dependence of the useful component on time $s(t_k) = s_k$ is unknown. The law of distribution of additive noise $\eta(t_k) = \eta_k$ is also limited, but it is assumed that the distribution density has a Gaussian law, with zero mathematical expectation and constant variance [8].

To conduct test studies, we will use the signals component models: rectangular, harmonic, triangular, parabolic and exponential.

3 Methods for Smoothing Digital Signals Based on Combined Criteria

The derivation of the estimate $\overline{s}_k = \overline{s}\left(t_k\right)$ of the s_k value can be interpreted as a decrease in the additive noise variance η_k. In the paper it is proposed to reduce the dispersion of the measured process by decreasing the sum of the squares of the finite differences of its values:

$$\sum_{k=1}^{n-1}\left(\overline{s}_k - \overline{s}_{k+1}\right)^2\,, \tag{2}$$

In this case, as a measure of the divergence of the input signal and its evaluation, the sum is used:

$$\sum_{k=1}^{n}\left(\overline{s}_k - Y_k\right)^2\,. \tag{3}$$

To determine the estimates of $\overline{s}_k$, we simultaneously, reduce the sums (2) and (3). This goal is achieved by minimizing two-criteria target functions of the form [10, 11]:

$$\varphi(\overline{s}_1,\overline{s}_2,...,\overline{s}_n)=\alpha\sum_{k=1}^{n}(\overline{s}_k-Y_k)^2+\sum_{k=1}^{n-1}(\overline{s}_k-\overline{s}_{k+1})^2\ ,\tag{4}$$

where α – constant coefficients.

It should be noted that the objective function (4) are continuous and bounded of below on the set R^n, therefore, at least at one point in the interval, the $(\overline{s}_1,\overline{s}_2,...,\overline{s}_n)$ reaches its lowest value. Let us prove the uniqueness of such a point, of objective function of the form (4). Due to the necessary condition of the extremum, its coordinates must satisfy the system of equations:

$$\frac{\partial\varphi}{\partial\overline{s}_j}=0,\ \ j=\overline{1,n}\,.\tag{5}$$

Of the following system of n linear equations with n unknowns $\overline{s}_1,\overline{s}_2,...,\overline{s}_n$:

$$\begin{cases}(1+\alpha)\overline{s}_1-\overline{s}_2-\alpha Y_1=0;\\ ....\\ (2+\alpha)\overline{s}_k-\overline{s}_{k+1}-\overline{s}_{k-1}-\alpha Y_k=0, \qquad k=2,3,...,n-1;\\ ...\\ (1+\alpha)\overline{s}_n-\overline{s}_{n-1}-\alpha Y_n=0.\end{cases}\tag{6}$$

We rewrite system (6) in the form:

$$\begin{cases}\overline{s}_2=(1+\alpha)\overline{s}_1-\alpha Y_1;\\ ...\\ \overline{s}_{k+1}=(2+\alpha)\overline{s}_k-\overline{s}_{k-1}-\alpha Y_k, \qquad k=2,3,...,n-1;\\ ...\\ (1+\alpha)\overline{s}_n-\overline{s}_{n-1}-\alpha Y_n=0.\end{cases}\tag{7}$$

Let us prove that the system of equations (7) has a unique solution. To this end, by the method of mathematical induction, we establish the validity of the assertion P_k: «the first $(k-1)$ arguments of systems (7) set variable $\overline{s}_1,\overline{s}_2,...,\overline{s}_k$, as linear functions of the argument $\overline{s}_1$, etc. $\overline{s}_j=\alpha_j\overline{s}_1+\beta_j$, when $\alpha_{j-1}<\alpha_j$, $j=\overline{1,k}$» at each $k=\overline{1,n}$ (use here $\alpha_0=0$). When $k=1$ we have the result $\alpha_1=1\ (>0)$, $\beta_1=0$, when $k=2$ – $\overline{s}_2=\alpha_2\overline{s}_1+\beta_2$, where $\alpha_2=1+\alpha>\alpha_1$, $\beta_2=-\alpha Y_1$, thus the statements P_1, P_2 are correct. Assuming the validity of the statement P_k for some $2\le k<n$, we prove the assertion P_{k+1}. From any k equation of the system (7) we obtain:

$$\overline{s}_{k+1}=(2+\alpha)(\alpha_k\overline{s}_1+\beta_k)-(\alpha_{k-1}\overline{s}_1+\beta_{k-1})-\alpha Y_k=\alpha_{k+1}\overline{s}_1+\beta_{k+1},$$

where $\alpha_{k+1}=(2+\alpha)\alpha_k-\alpha_{k-1}>\alpha_k$; $\beta_{k+1}=(2+\alpha)\beta_k-\beta_{k-1}-\alpha Y_k$.

The above calculations for $P_1,P_2,\ldots,P_n$ are correct. Using the expression P_n , the last equation of the system (10) reduces to the form $\gamma\overline{s}_1+\delta=0$, where $\gamma=(1+\alpha)\alpha_n-\alpha_{n-1}=(\alpha_n-\alpha_{n-1})+\alpha\cdot\alpha_n>0$, $\delta=(1+\alpha)\beta_n-\beta_{n-1}-\alpha Y_n$. The resulting equation has a unique solution $\overline{s}_1=-\delta/\gamma$, in which the values $\overline{s}_k=\alpha_k\overline{s}_1+\beta_k$, where $k=\overline{2,n}$.

Thus, the system of equations (4) has a unique solution.

We also propose the definition of an exact analytic solution of a two-criteria objective function of the form (4). As proved, the minimum point of the function (4) is the unique solution of the system of linear equations (6).

We show that this solution has the form:

$$\overline{s}_k=\gamma_k\overline{s}_1-\sum_{i=1}^{k-1}y_i\beta_{k-i}\ ,\quad k=1,2,\ldots,n\ , \tag{8}$$

where

$$\gamma_k=\sum_{j=0}^{k-1}\binom{k+j-1}{2j}\alpha^j\ , \tag{9}$$

$$\beta_k=\sum_{j=1}^{k}\binom{k+j-1}{2j-1}\alpha^j\ . \tag{10}$$

Note. Here and in what follows we establish the notation for binomial coefficients

$$\binom{m}{l}=\begin{cases}0, & l<0,\\ 1, & l=0,\\ \dfrac{m\cdot(m-1)\cdot\ldots\cdot(m-l+1)}{l!}, & l>0.\end{cases}$$

$$\overline{s}_1=\frac{\alpha\sum_{i=1}^{n}\gamma_{n-i+1}y_i}{\beta_n}\ . \tag{11}$$

Using the relations (8) and (9) for $k=2$ and the relation (10) for $k=1$, we obtain:

$$\overline{s}_2=\gamma_2\overline{s}_1-y_1\beta_1=\left(\binom{1}{0}+\alpha\binom{2}{2}\right)\overline{s}_1-y_1\alpha\binom{1}{1}=(1+\alpha)\overline{s}_1-y_1\alpha\ .$$

Substituting the result in the first equation (6), we obtain the identity:

$$(1+\alpha)\overline{s_1} - ((1+\alpha)\overline{s_1} - \alpha y_1) = \alpha y_1 .$$

Let us verify that the quantities (8) (under the conditions (9) – (11)) satisfy any equation of the system (6) and with $2 \le k \le n-1$. Thus,

$$(2+\alpha)\left(\sum_{j=0}^{k-1}\binom{k+j-1}{2j}\alpha^j\overline{s_1} - \nu_k\right) - \left(\sum_{j=0}^{k-2}\binom{k+j-2}{2j}\alpha^j\overline{s_1} - \nu_{k-1}\right) -$$

$$-\left(\sum_{j=0}^{k}\binom{k+j}{2j}\alpha^j\overline{s_1} - \nu_{k+1}\right) = \alpha y_k ,$$

where

$$\nu_k = \sum_{i=1}^{k-1} y_i \beta_{k-i} . \tag{12}$$

We transform the left side of the equation:

$$\left(\left(2+\alpha\right)\alpha^{k-1} + \sum_{j=0}^{k-2}\left((2+\alpha)\binom{k+j-1}{2j} - \binom{k+j-2}{2j} - \binom{k+j}{2j}\right)\alpha^j -\right.$$

$$\left. - \alpha^k - (2k-1)\alpha^{k-1}\right)\overline{s_1} - (2+\alpha)v_k + v_{k-1} + v_{k+1} = \alpha y_k .$$

We simplify part of the expression on the left side, using the properties of binomial coefficients [9]:

$$\sum_{j=0}^{k-2}\left((2+\alpha)\binom{k+j-1}{2j} - \binom{k+j-2}{2j} - \binom{k+j}{2j}\right)\alpha^j =$$

$$= \sum_{j=0}^{k-2}\left(2\binom{k+j-1}{2j} + \alpha\binom{k+j-1}{2j} - \binom{k+j-2}{2j} - \binom{k+j}{2j}\right)\alpha^j =$$

$$= \sum_{j=0}^{k-2}\left(2\binom{k+j-1}{2j} - \binom{k+j-2}{2j} - \binom{k+j}{2j}\right)\alpha^j + \sum_{j=0}^{k-2}\alpha\binom{k+j-1}{2j}\alpha^j =$$

$$= 2\binom{k-1}{0} - \binom{k-2}{0} - \binom{k}{0} + \binom{2k-3}{2k-4}\alpha^{k-1} +$$

$$+ \sum_{j=1}^{k-2}\left(2\binom{k+j-1}{2j} + \binom{k+j-2}{2j-2} - \binom{k+j-2}{2j} - \binom{k+j}{2j}\right)\alpha^j =$$

$$=(2k-3)\alpha^{k-1}+$$

$$+\sum_{j=1}^{k-2}\left(\binom{k+j-1}{2j}+\binom{k+j-1}{2j}+\binom{k+j-2}{2j-2}-\binom{k+j-2}{2j}-\binom{k+j}{2j}\right)\alpha^{j}=$$

$$=(2k-3)\alpha^{k-1}+\sum_{j=1}^{k-2}\left(\binom{k+j-2}{2j-1}-\binom{k+j-1}{2j-1}+\binom{k+j-2}{2j-2}\right)\alpha^{j}=$$

$$=(2k-3)\alpha^{k-1}+\sum_{j=1}^{k-2}\left(\binom{k+j-2}{2j-2}-\binom{k+j-2}{2j-2}\right)\alpha^{j}=(2k-3)\alpha^{k-1}.$$

(Here and in what follows we assume that when $l>m$ sum $\sum_{j=l}^{m}b_j$ is zero).

Thus, any average equation of system (6) takes the form:

$$(0)\overline{s}_1-(2+\alpha)v_k+v_{k-1}+v_{k+1}=\alpha y_k.$$

Taking into account expressions (10) and (12), the resulting ratio is rewritten as follows:

$$-(2+\alpha)\sum_{i=1}^{k-1}y_i\sum_{j=1}^{k-i}\binom{k+j-i-1}{2j-1}\alpha^{j}+\sum_{i=1}^{k-2}y_i\sum_{j=1}^{k-i-1}\binom{k+j-i-2}{2j-1}\alpha^{j}+$$

$$+\sum_{i=1}^{k}y_i\sum_{j=1}^{k-i+1}\binom{k+j-i}{2j-1}\alpha^{j}=\alpha y_k.$$

We simplify the left-hand side of the equation using the properties of binomial coefficients:

$$-(2+\alpha)\alpha y_{k-1}-(2+\alpha)\sum_{i=1}^{k-2}y_i\sum_{j=1}^{k-i}\binom{k+j-i-1}{2j-1}\alpha^{j}+$$

$$\sum_{i=1}^{k-2}y_i\sum_{j=1}^{k-i-1}\binom{k+j-i-2}{2j-1}\alpha^{j}+\alpha y_k+y_{k-1}(2\alpha+\alpha^2)+$$

$$+\sum_{i=1}^{k-2}y_i\sum_{j=1}^{k-i+1}\binom{k+j-i}{2j-1}\alpha^{j}=\alpha y_k-(2+\alpha)\sum_{i=1}^{k-2}y_i\alpha^{k-i}+$$

$$+\sum_{i=1}^{k-2}y_i\left(\alpha^{k-i+1}+(2k-2i)\alpha^{k-i}\right)-$$

$$-\sum_{i=1}^{k-2}y_i\sum_{j=1}^{k-i-1}\left((2+\alpha)\binom{k+j-i-1}{2j-1}-\binom{k+j-i-2}{2j-1}-\binom{k+j-i}{2j-1}\right)\alpha^{j}=$$

$$=\alpha y_k+\sum_{i=1}^{k-2} y_i(2k-2i-2)\alpha^{k-i}-$$

$$-\sum_{i=1}^{k-2} y_i\sum_{j=1}^{k-i-1}\left((2+\alpha)\binom{k+j-i-1}{2j-1}-\binom{k+j-i-2}{2j-1}-\binom{k+j-i}{2j-1}\right)\alpha^j.$$

We will convert the coefficient at y_i in the last sum:

$$\sum_{j=1}^{k-i-1}\left(2\binom{k+j-i-1}{2j-1}+\alpha\binom{k+j-i-1}{2j-1}-\binom{k+j-i-2}{2j-1}-\binom{k+j-i}{2j-1}\right)\alpha^j=$$

$$=\sum_{j=1}^{k-i-1}\left(\binom{k+j-i-1}{2j-1}+\binom{k+j-i-1}{2j-1}-\binom{k+j-i-2}{2j-1}-\binom{k+j-i}{2j-1}\right)\alpha^j+$$

$$+\sum_{j=1}^{k-i-1}\binom{k+j-i-1}{2j-1}\alpha^{j+1}=\sum_{j=1}^{k-i-1}\left(\binom{k+j-i-2}{2j-2}-\binom{k+j-i-1}{2j-2}\right)\alpha^j+$$

$$+\sum_{j=2}^{k-i}\binom{k+j-i-2}{2j-3}\alpha^j=(2k-2i-2)\alpha^{k-i}+\sum_{j=2}^{k-i-1}\binom{k+j-i-2}{2j-3}\alpha^j-$$

$$-\sum_{j=2}^{k-i-1}\binom{k+j-i-2}{2j-3}\alpha^j=(2k-2i-2)\alpha^{k-i}.$$

Thus, any average equation of system (6) becomes an sameness:

$$\alpha y_k+\sum_{i=1}^{k-2} y_i(2k-2i-2)\alpha^{k-i}-\sum_{i=1}^{k-2} y_i(2k-2i-2)\alpha^{k-i}=\alpha y_k,$$

$$k=2,3,...,n-1.$$

Let us prove that the quantities (8) satisfy the last equation of the system (6):

$$(1+\alpha)\left(\sum_{j=0}^{n-1}\binom{n+j-1}{2j}\alpha^j\overline{s_1}-v_n\right)-\left(\sum_{j=0}^{n-2}\binom{n+j-2}{2j}\alpha^j\overline{s_1}-v_{n-1}\right)=\alpha y_n,$$

or

$$\left((1+\alpha)\alpha^{n-1}+\sum_{j=0}^{n-2}\left((1+\alpha)\binom{n+j-1}{2j}-\binom{n+j-2}{2j}\right)\alpha^j\right)\overline{s_1}-$$

$$-(1+\alpha)v_n+v_{n-1}=\alpha y_n.$$

The coefficient at $\overline{s_1}$ is

$$(1+\alpha)\alpha^{n-1}+\sum_{j=0}^{n-2}\left(\binom{n+j-1}{2j}-\binom{n+j-2}{2j}\right)\alpha^j+\sum_{j=1}^{n-1}\binom{n+j-2}{2j-2}\alpha^j=$$

$$=(1+\alpha)\alpha^{n-1}+\sum_{j=1}^{n-2}\binom{n+j-2}{2j-1}\alpha^{j}+\sum_{j=1}^{n-1}\binom{n+j-2}{2j-2}\alpha^{j}=$$

$$=(1+\alpha)\alpha^{n-1}+(2n-3)\alpha^{n-1}+\sum_{j=1}^{n-2}\binom{n+j-1}{2j-1}\alpha^{j}=\sum_{j=1}^{n}\binom{n+j-1}{2j-1}\alpha^{j}=\beta_{n}.$$

The equation takes the form

$$\beta_{n}\overline{s}_{1}-\sum_{i=1}^{n-1}y_{i}\left(\sum_{j=1}^{n-i}(1+\alpha)\binom{n+j-i-1}{2j-1}\alpha^{j}-\sum_{j=1}^{n-i-1}\binom{n+j-i-2}{2j-1}\alpha^{j}\right)=\alpha y_{n}.$$

The expression in brackets is

$$\sum_{j=2}^{n-i}\binom{n+j-i-2}{2j-3}\alpha^{j}+\sum_{j=1}^{n-i-1}\left(\binom{n+j-i-1}{2j-1}-\binom{n+j-i-2}{2j-1}\right)\alpha^{j}+(1+\alpha)\alpha^{n-i}=$$

$$=\alpha^{n-i+1}+\alpha+\sum_{j=2}^{n-i}\left(\binom{n+j-i-2}{2j-2}+\binom{n+j-i-2}{2j-3}\right)\alpha^{j}=$$

$$=\alpha^{n-i+1}+\sum_{j=1}^{n-i}\binom{n+j-i-1}{2j-2}\alpha^{j}=\alpha\sum_{j=0}^{n-i}\binom{n+j-i}{2j}\alpha^{j}=\alpha\gamma_{n-i+1}.$$

Because $\gamma_{1}=1$, to $\beta_{n}\overline{s}_{1}-\alpha\sum_{i=1}^{n}y_{i}\gamma_{n-i+1}=0$, and this equality is satisfied, in view of (11).

Thus, the expression (8) (when the expressions (9) − (11) is substituted into it) has a unique solution of the system of equations (6), which minimizes the function (4). It should be noted that this function does not have other minimum points.

To test the effectiveness of a multi-criteria method for smoothing digital signals, the standard deviation σ_{o} of the estimates from the values of the input signal is used as a criterion.

Fig. 1 shows the dependencies of the root-mean-square deviation on the values of the coefficient α obtained by processing a two-criteria objective function, with the analytical solution and the iterative approach. As a useful component (1), a function is used whose discrete values are described by a parabola, with a root-mean-square noise deviation σ_{n} = 0.15 [9].

The analysis of the curves presented in Fig. 1 allows us to conclude that the results of the processing efficiency estimates obtained by the iterative algorithm (4) and the analytical solution practically coincide, deviation σ_{o} is less than 1%.

Investigations of the effect of coefficients on the result are presented in [10], ranges of confidence intervals are shown coefficient α. To minimize the value of the root-mean-square error, it is necessary to optimize the choice of the approximation ε. To determine the minimum error and to determine the effect on the error parameter ε, we fix the parameter $\alpha_{\min}$ when $\sigma_o(\alpha_{\min}) \to \min$.

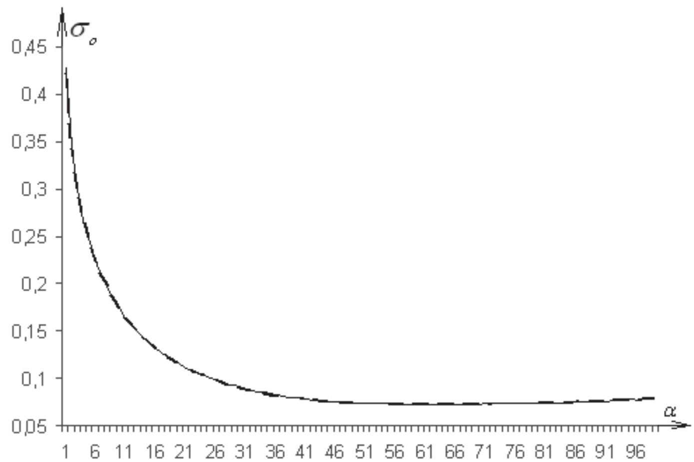

Fig. 1 – *Dependence of* $\sigma_o(\alpha)$ *for the analytical and iterative solution.*

As a result of simulation modeling, for various models of the useful component of functions, we the dependence of the parameter change $\sigma_o(\varepsilon)$. The value $\sigma_o(\varepsilon)$ of the root-mean-square error obtained when processing the input realization, for fixed values of the standard deviation of the additive noise component, is coincided. Fig. 2 shows the graphs of the change $\sigma_o(\varepsilon)$, in the case of using a harmonic signal with different mean square deviation of the noise component.

The results of simulation presented in Fig. 2 make it possible to conclude that the values of the smoothing error $\sigma_o(\varepsilon)$ at which $\sigma_o(\alpha_{\min}) \to \min$ lies at the point $\varepsilon = 0.001$.

In order to determine the required computationally-time costs for the implementation of the proposed method, we estimate the number of required m iterations, in order to achieve the approximation parameter of the estimates of $\varepsilon = 0.001$ for fixed values of $\alpha_{\min}$. During the simulation, data were obtained showing that the number of iterations depends on the root-mean-square deviation of the noise σ_n and the shape of the useful component. **Table 1** shows the values of m obtained with the value of $\varepsilon_{\min}$ in the case when the condition for ensuring a given accuracy is achieved when solving the problem of smoothing by multi-criteria method of smoothing digital signals in conditions of limited volume of a priori information.

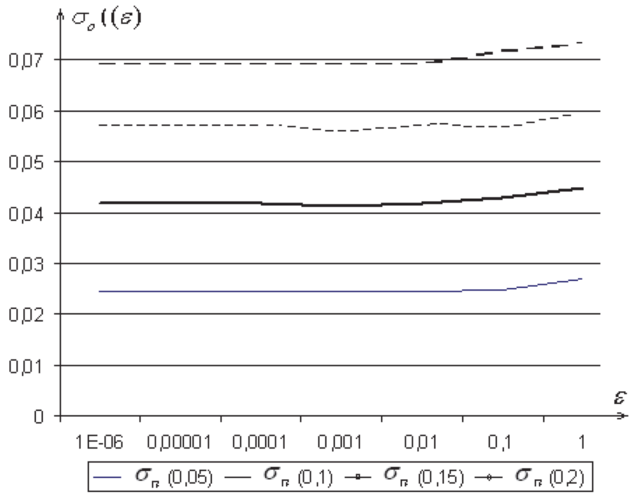

Fig. 2 – *Schedule for changing the parameter* $\sigma_o(\varepsilon)$.

Table 1
The number of iterations m required to perform the smoothing operation as a function depending the useful and noise components.

σ_n	signal / m	The sin function	The exponent function	The parabolic function	The composite function	The square pulse
0,05	m with $\alpha_{\min 1}$	20	40	37	21	5
0,1	m with $\alpha_{\min 1}$	35	61	57	39	9
0,15	m with $\alpha_{\min 1}$	45	91	73	50	14
0,2	m with $\alpha_{\min 1}$	57	135	103	89	24

Fig. 3 shows an example of the determination of the number of iterations $m(\varepsilon)$, at the input of the filter a mixture of sine and noise with $\sigma_n = 0.1$.

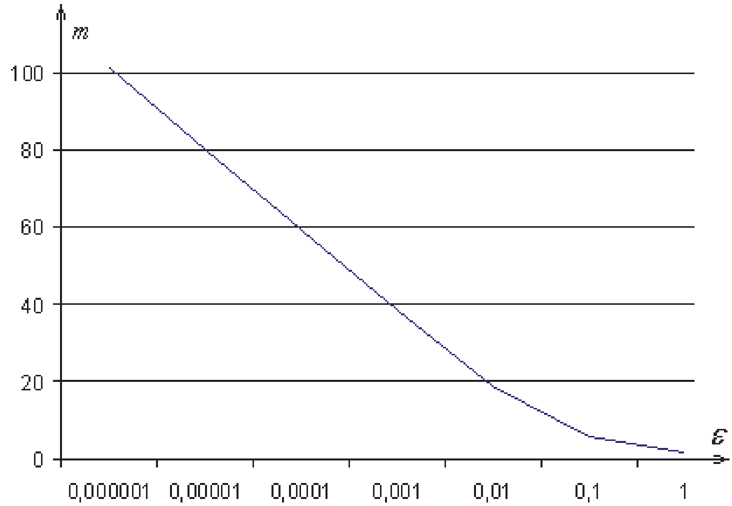

Fig. 3 – *An example of calculating the number of iterations.*

The results shown in Fig. 3 show that as the value of ε increases, the number of required repetitions of the calculation decreases.

4 Testing Results

We analyze on a set of test signals of the form: the sin function, the exponent function, the parabolic function, composite signal, square pulse. We compared the method with the standard implementations of signal processing methods (weighted moving average, exponential smoothing, and median smoothing) presented in the Matlab library. The results of the root-mean-square error σ_o obtained are presented in **Table 2**. The results are averaged over a thousand implementations.

Table 2

Results of mean square error σ_o.

Amplitude noise / method	$\sigma_n = 0.05$	$\sigma_n = 0.1$	$\sigma_n = 0.15$	$\sigma_n = 0.2$
weighted moving average	0,025	0,041	0,068	0,085
exponential smoothing	0.38	0.67	0.084	0.121
median smoothing	0,038	0,064	0,075	0,084
two-criteria method	0,024	0,041	0,055	0,069

The method of the weighted moving average is the closest to the developed method in the case of small values of the root-mean-square deviation of noise. Most methods work in the presence of a priori information about the function of the useful signal, otherwise the root-mean-square error can dramatically increase on 40% or more.

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