The Mathematical Model for Analysis and Evaluation of the Transient Process of the three–phase Asynchronous Machine Performance

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Abstract: This paper describes an adopted method which could be used in transient process modelling of three–phase asynchronous machines on the computer. Converter asynchronous electric machine feeding requires a detailed research of the transient process influence and the dynamic states of energy exchange between the electrical circuit feeding and the machine itself. A mathematical model has been formulated in order to solve the transient process of the three-phase asynchronous electric machine referring to the two asynchronous machines and containing two matrix models defined by Park's $d-q$ components. In the system of symmetrical components the state and the operation of the first machine was defined based on relations and operation conditions of a direct sequence system and of other machines based on relations and operation conditions of an inverse sequence system. The new method has been derived from the existing `psbpwm` MATLAB package. The new model simulates the state and operation of a single machine with identical parame-ters through the state and operation of the two machines with identical parameters: the first in a direct sequence system and the others in an inverse sequence system. Operation analysis of the three–phase induction machine is limited to a transient process occurring in the right angled shape of voltage feeding. The new presented model can, with certain limitations, be applied not only to a three–phase asynchro-nous machine model but to other alternating current machines which have a stator and a rotor. The experiment has been performed following an example of asym-metrical feeding followed by inverse component phenomena of electrical and mechanical values in the machine.

Keywords: Mathematical Model, One-phase Interruption, Asymmetric Mode, Three-Phase Asynchronous Machine.

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1   Introduction

Voltage symmetry (amplitude and phase) is one of the characteristics of voltage based on the criteria of electric energy quality. IEC and EN 50160 standards define four characteristics of voltage with regard to its values, and those are: a) frequency, b) amplitude, c) waveform, d) three phase network symmetry [1]. At an asynchronous machine starting, phase windings do not switch on simultaneously thereby creating a special state that must be represented through special electric circuits related to the disturbance state. An asynchronous machine starting using switches is a process in which each phase switches on at a different time which represents a phase current time–form coordinate [2].

Three–phase network performance analysis or analysis of one of its elements in symmetrical regimes is based on the analysis of equivalent single–phase networks. Transient scheme simulation in MATLAB–Simulink which contains important element parameters, can be used to analyse electrical (voltage, current) and mechanical (moment, speed) properties of transient states and determine the level of their influence on functional and protective element performance in networks/installation. Otherwise, in the case of asymmetrical network faults, the system results in asymmetric loading, so the network cannot be reduced to a single phase [3, 4].

Asymmetries could occur in cases of voltage unbalance or electrical impedance GDS (General Distributive System) [1], (due to disturbances, a single–phase load changes in GDS, phase cutoff or feeding disturbances from converters). GDS according to IEC60364–1 standard is defined as a feeding system within the network/installation which determines: general system connection to ground, individual connection of exposed conductive parts to ground and the relation between the neutral and protective conductors. IEC standard subdivides disturbances which occur in three–phase electrical GDS networks, into faults and interferences. Faults (cross asymmetries) are: short circuit: single–phase, two–phase, two–phase ground faults, three–phase and ground connections. Interferences (vertical asymmetries) are: asymmetrical loads and phase cutoffs, phase loads and voltage changes, frequency and direction of power flow changes. Permanently simplified analysis is not possible due to the fact that phase components cannot be determined solely according to GDS equations containing cyclic impedance of elements (phase impedance of conductors is no longer equal) and application of Ohm's and Kirchhoff's law becomes complex and demanding. The method of symmetrical components, is based on Thévenin's theorem and the principle of superposition in linear systems and it is known as one of the vector methods; it is very important for the three–phase electrical networks with linear parameters and identical frequency [5, 6].

2   The Method of Symmetrical Components

Three balanced electromotive forces (EMF) of the direct sequence system affect the symmetrical regime with linear (GDS), composed of identical impedances in all three phases [7]. EMF of an inverse and zero sequence system equal to zero, but in the case of a changed GDS structure, voltage and currents transfer to asymmetrical domain. Based on the criteria of superposition, the GDS state is described through the states of the three internal systems: positive, inverse and zero sequence [7, 8]. In GDS`s, with linear parameters (symmetrical GDS), system currents are independently connected to voltages of the some system, but the impedance values depend on the system structure. Impedance of direct Z_d, inverse Z_i and zero sequence Z_0 depend on real independent phase impedance of elements and partially on mutual inductance and capacitance.

For the GDS which contains only the EMF of a direct sequence, symmetrical components of voltage and currents are respectively: V_d, V_i, V_0, I_d, I_i, I_0, in asymmetrical domain, and relations which define this state are:

$$E = V_d + Z_d I_d, \ 0 = V_i + Z_i I_i, \ 0 = V_0 + Z_0 I_0. \tag{1}$$

Equations (1) which correspond to the GDS with multiple sources, which produce the EMF, and correspond respectively to internal impedances Z_d, Z_i and Z_0 are derived based on the Thévenin's theorem on equivalent generators.

Mathematical model is a system of differential equations and corresponding algebraic expressions. Equations which connect symmetrical components of GDS voltage, EMF and voltage decreases in windings of the three–phase asynchronous machines depend on whether there is a cross or vertical asymmetry of elements:

$$I_A \neq 0, \quad I_B \neq 0, \quad I_C \neq 0, \quad \text{and/or}$$
$$V_A \neq 0, \quad V_B \neq 0, \quad V_C \neq 0, \quad \text{and/or} \tag{2}$$
$$V_A \neq V_B \neq V_C, \quad I_A \neq I_B \neq I_C.$$

For cross asymmetries additional conditions apply:

$$0 = V_d' - z_d' I_d', \quad 0 = V_i' - z_i' I_i', \quad 0 = V_0' - z_0' I_0', \quad I_d \neq I_d', \quad I_i \neq I_i', \quad I_0 \neq I_0'. \tag{3}$$

Cross–asymmetry is created when one phase is excluded (for example. In the case of phase A cutoff), and this entails that the phase conductor is cutoff between the two points when equivalent circuits of inverse, zero and direct sequence are established. z_d, z_i, z_0 are circuit impedance on the left side from the point of cutoff toward the GDS, and z_d', z_i', z_0' are circuit impedance on the right side from the point of cutoff toward the asynchronous machine. Values, at the point of cutoff, are: voltage values V_A, V_B, V_C, currents I_A, I_B, I_C, impedance

z_A, z_B, z_C on the side of GDS, phase impedance on the load side (of the machine) z'_d, z'_i, z'_0 and electrical scheme which relates to these values. These values are included in equations (5) of phase voltage V_A, V_B, V_C and currents I_A, I_B, I_C and with the help of transformation matrix are included in relations of symmetrical components. Transformation from asymmetrical three–phase system to a symmetrical system is performed based on [7, 8]:

$$UNS_3(\overline{V}) = S(\overline{V}_0, \overline{V}_d, \overline{V}_i), \quad a = e^{j120^o}, \quad (1 + a + a^2) = 0. \tag{4}$$

In a three–phase system, phase vectors are $\overline{V}_A$, $\overline{V}_B$, $\overline{V}_C$ and component vectors are $\overline{V}_d$, $\overline{V}_i$, $\overline{V}_0$.

Transformation matrix $\|S\|$ from real phase to symmetrical components is:

$$\|F_{d,i,0}\| = \|S\| \cdot \|F_{A,B,C}\|, \quad \|F_{A,B,C}\| = \begin{Vmatrix} V_A \\ V_B \\ V_C \end{Vmatrix}, \quad \|F_{d,i,0}\| = \begin{Vmatrix} V_d \\ V_i \\ V_0 \end{Vmatrix}, \tag{5}$$

$$\|S\| = \frac{1}{3} \begin{Vmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & a^2 & 1 \end{Vmatrix}, \quad \|S\|^{-1} = \begin{Vmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{Vmatrix}, \tag{6}$$

$$\begin{Vmatrix} F_A \\ F_B \\ F_C \end{Vmatrix} = \|S\|^{-1} \|F_{d,i,0}\| = \begin{Vmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} F_d \\ F_i \\ F_0 \end{Vmatrix}, \tag{7}$$

$$\begin{Vmatrix} F_d \\ F_i \\ F_0 \end{Vmatrix} = \|S\| \cdot \|F_{A,B,C}\| = \frac{1}{3} \begin{Vmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & a^2 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} F_A \\ F_B \\ F_C \end{Vmatrix}. \tag{8}$$

$\|S\|^{-1}$ denote the transformation matrix Table
rom symmetrical components to real phase components:

$$\|F_{A,B,C}\| = \|S\|^{-1} \|F_{d,i,0}\|, \tag{9}$$

$$\overline{V}_A = \overline{V}_d + \overline{V}_i + \overline{V}_0, \quad I_A = I_d + I_i + I_0,$$
$$\overline{V}_B = a^2 \overline{V}_d + a \overline{V}_i + \overline{V}_0, \quad I_B = a^2 I_d + a I_i + I_0, \tag{10}$$
$$\overline{V}_C = a \overline{V}_d + a^2 \overline{V}_i + \overline{V}_0, \quad I_C = a I_d + a^2 I_i + I_0.$$

The direction of the vector source voltage rotation is anti–clockwise rotation and according to equation (5) symmetrical components of a three–phase system is determined as:

$$\overline{V}_d = \frac{1}{3}\Big[\ \overline{V}_A + a\overline{V}_B + a^2\overline{V}_C\ \Big],$$

$$\overline{V}_i = \frac{1}{3}\Big[\ \overline{V}_A + a^2\overline{V}_B + a\overline{V}_C\ \Big],\ . \tag{11}$$

$$\overline{V}_0 = \frac{1}{3}\Big[\ \overline{V}_A + \overline{V}_B + \overline{V}_C\ \Big].$$

The expressions in (11) relate to the voltage V, where these mathe-matical operations equally apply to currents I.

Asymmetrical three–phase voltage and currents are determined by transformation to a symmetrical component system. The superposition principle is applicable only if electrical parameters are simple periodical [9]. Dividing the right angled shape of voltage feeding function into the basic one and multi harmonics is realised through block Fourier `psbpwm` MATLAB Simulink package [10, 11].

Voltages at the machine ends are symmetrical, if the machine is connected to the voltage system in the GDS, and symmetry can be disturbed by a phase cutoff or feeding from convertors [6, 12]. The form of symmetrical components from equation (11) can for some reason be modified, for example, to solve simultaneous asymmetries on GDS models or to introduce the new system components [10, 13] (transformations: Edith Clarke – α, β, 0, Park – d, q, 0, Kogo – r, s, t, etc.).

3 Analysis of the Existing Asynchronous Machine Model in MATLAB Simulink

The two mathematical models of asynchronous machines are recognized, depending on their application [10]:

1. (Edith) Clarke – fixed model in unique "α – β" coordinate system, and $\omega t = 0$ connected to stator with Clarke's transformation matrix. The machine model in the so-called, stator coordinate system is convenient for machine performance analysis because all its parameters in a stationary state are simple periodical, owing to invariance of amplitudes, and, additionally, are equal to real phase parameters in all the regimes.

2. Mobile Park mathematical model – synchronous $d – q$ model in which $\omega t \neq 0$. The system matrix $0dq$ best suits the machine structure, and takes into consideration impedance inequality of cross and vertical axis. Parks transformation matrix determines a mathematical model in a

synchronous coordinate system and is used for transfer process analysis in electrical machines [5, 14].

Fig. 1 presents schemes with the process in a single asynchronous machines being equivalent to the processes in two asynchronous machines out of which the first one operates in a system of direct, Fig. 1a, and the second, Fig. 1b in a system of inverse sequence (a symmetrical system with *d*, *i*, 0 components).

(a)

(b)
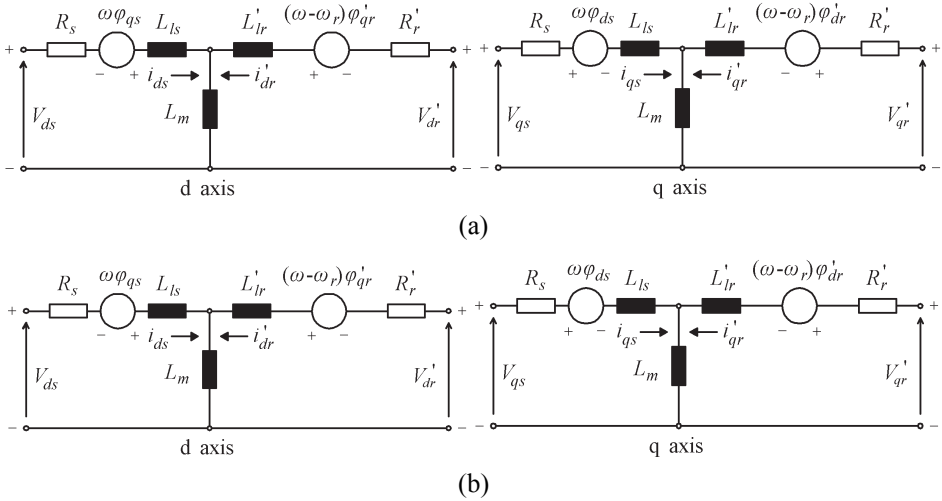
Fig. 1 – *The asynchronous machine model whose performance is presented using the processes of the two machines, out of which, the first operates in a direct system and the second in an inverse sequence system (a symmetrical system with d, i, 0 components): a) Machine positive sequence ω positive seq =| ω |= 2πf ; b) Machine negative sequence ω negative seq = − | ω |= −2πf .*

The existing asynchronous machine model in MATLAB Simulink, contains special blocks used to specify the electrical and mechanical p.u. parameters of the model): R_s, L_{ls} Stator resistance and leakage inductance; R_r', L_{lr}' Rotor resistance and leakage inductance; L_m Magnetizing inductance; L_s, L_r' Total stator and rotor inductance; V_{qs}, i_{qs}, *q* axis stator voltage and current; V_{qr}', i_{qr}', *q* axis rotor voltage and current; V_{ds}, i_{ds} *d* axis stator voltage and current; V_{dr}', i_{dr}' *d* axis rotor voltage and current; φ_{qs}, φ_{ds} Stator *q* and *d* axis flux; φ_{qr}', φ_{dr}' Rotor *q* and *d* axis flux; ω_m Angular velocity of the rotor; θ_m Rotor angular position; *p* Number of pole pairs; $\omega_m \times p = \omega_r$ Electrical angular velocity; $\theta_m \times p = \theta_r$ Electrical angular velocity; T_e Electromagnetic torque angular velocity; T_m Shaft mechanical torque angular velocity; *J* Combined rotor and

load inertia coefficient. Set to infinite to simulate locked rotor; H combined rotor and load constant. Set to infinite to simulate locked rotor; F combined rotor and load viscous friction coefficient.

The existing mathematical model included in the algorithm of the asynchronous machine dynamic performance model corresponds to Machine 1, Fig. 4, which basically operates in a system of a direct sequence (d) and feeds from convertors of in between phase voltage V_{AB}. All simple periodical values in a system have a cyclic frequency change $\omega = 2\pi f$, $\theta = \omega t = 2\pi ft$ which means that the following equation applies to Machine 1: $\omega = +2\pi f = \omega_d$, $\theta_d = \omega t = +2\pi ft$. The positive sign (+) coincides with the vector rotation of a direct sequence system (''positive sequence'').

An electrical part of the machine is represented as being fourth end and the mechanical as a second order system. Electrical variables are reduced to stator. Presented using symbol (') in the following equations. Electrical circuit equations from Fig. 1a: Stator and rotor voltages reduced to stator are:

$$\begin{Vmatrix} V_{ds} \\ V_{qs} \end{Vmatrix} = R_s \begin{Vmatrix} i_{ds} \\ i_{qs} \end{Vmatrix} + \frac{\mathrm{d}}{\mathrm{d}t} \begin{Vmatrix} \varphi_{ds} \\ \varphi_{qs} \end{Vmatrix} + \omega \begin{Vmatrix} 0 & -1 \\ 1 & 0 \end{Vmatrix} \begin{Vmatrix} \varphi_{ds} \\ \varphi_{qs} \end{Vmatrix},$$
$$\begin{Vmatrix} V'_{dr} \\ V'_{qr} \end{Vmatrix} = R'_r \begin{Vmatrix} i'_{dr} \\ i'_{qr} \end{Vmatrix} + \frac{\mathrm{d}}{\mathrm{d}t} \begin{Vmatrix} \varphi'_{dr} \\ \varphi'_{qr} \end{Vmatrix} + (\omega - \omega_r) \begin{Vmatrix} 0 & -1 \\ 1 & 0 \end{Vmatrix} \begin{Vmatrix} \varphi'_{dr} \\ \varphi'_{qr} \end{Vmatrix}.$$

(12)

Inductance of stator and rotor reduced to rotor is:

$$L_s = L_{ls} + L_m, \quad L'_r = L'_{lr} + L_m.$$

(13)

The magnetic circuit which relates to included fluxes is:

$$\varphi_{ds} = L_s i_{ds} + L_m i'_{dr}, \quad \varphi_{qs} = L_s i_{qs} + L_m i'_{qr},$$
$$\varphi'_{dr} = L'_r i'_{dr} + L_m i_{ds}, \quad \varphi'_{qr} = L'_r i'_{qr} + L_m i_{qs}.$$

(14)

The Mechanical parameter system is:

$$\frac{\mathrm{d}}{\mathrm{d}t}\theta = \omega, \quad \frac{\mathrm{d}}{\mathrm{d}t}\theta_m = \omega_m, \quad \frac{\mathrm{d}}{\mathrm{d}t}\omega_m = \frac{1}{2H}(T_e - F\omega_m - T_m).$$

(15)

The moment equation is:

$$T_e = \frac{3}{2}p(\varphi_{ds}i_{qs} - \varphi_{qs}i_{ds}).$$

(16)

Exploring the possibility of introducing Machine 2 to a model with changed parameters of cyclic frequency $\omega = -2\pi ft = \omega_i / -\omega = 2\pi ft = \omega_i$ and $\theta_i = -\omega t = -2\pi ft / -\theta_i = \omega t = -2\pi ft$ corresponds to vector rotation in a negative sequence system. The advantages of this approach are that Machine 2 can retain the some electrical and mechanical parameters which already exist in a mathematic-

cal model of the machine.

Electrical circuit equations from Fig. 1b: Stator and rotor voltages reduced to stator (instead of (+ω), (−ω) was introduced to a relation which corresponds to the relative rotation speed of the rotor in a system of inverse distribution) are:

$$\begin{Vmatrix} V_{ds} \\ V_{qs} \end{Vmatrix} = R_s \begin{Vmatrix} i_{ds} \\ i_{qs} \end{Vmatrix} + \frac{\mathrm{d}}{\mathrm{d}t} \begin{Vmatrix} \varphi_{ds} \\ \varphi_{qs} \end{Vmatrix} - \omega \begin{Vmatrix} 0 & -1 \\ 1 & 0 \end{Vmatrix} \begin{Vmatrix} \varphi_{ds} \\ \varphi_{qs} \end{Vmatrix},$$

$$\begin{Vmatrix} V'_{dr} \\ V'_{qr} \end{Vmatrix} = R'_r \begin{Vmatrix} i'_{dr} \\ i'_{qr} \end{Vmatrix} + \frac{\mathrm{d}}{\mathrm{d}t} \begin{Vmatrix} \varphi'_{dr} \\ \varphi'_{qr} \end{Vmatrix} + \left(-\omega - \omega_r \right) \begin{Vmatrix} 0 & -1 \\ 1 & 0 \end{Vmatrix} \begin{Vmatrix} \varphi'_{dr} \\ \varphi'_{qr} \end{Vmatrix}. \tag{17}$$

Stator and rotor inductance reduced to stator is:

$$L_s = L_{ls} + L_m, \quad L'_r = L'_{lr} + L_m. \tag{18}$$

The magnetic circuit which relates to included fluxes is:

$$\varphi_{ds} = L_s i_{ds} + L_m i'_{dr}, \quad \varphi_{qs} = L_s i_{qs} + L_m i'_{qr},$$
$$\varphi'_{dr} = L'_r i'_{dr} + L_m i_{ds}, \quad \varphi'_{qr} = L'_r i'_{qr} + L_m i_{qs}. \tag{19}$$

The mechanical parameter system: In a Machine 2 model a symbol on the left and the right side of the relation is changed:

$$\frac{\mathrm{d}}{\mathrm{d}t} \theta = -\omega, \quad -\frac{\mathrm{d}}{\mathrm{d}t} \theta_m = -\omega_m, \quad -\frac{\mathrm{d}}{\mathrm{d}t} \omega_m = -\frac{1}{2H} \left(T_e - F\omega_m - T_m \right). \tag{20}$$

The moment equation is:

$$-T_e = -\frac{3}{2} p \left(\varphi_{ds} i_{qs} - \varphi_{qs} i_{ds} \right). \tag{21}$$

Voltage, current and moment values will be positive on diagrams, due to the sign change on the left and the right side of the equation (17, 20 and 21). Rotor rotation of the machine will be negative due to the negative sign. This is logical as the Machine 2 operates in a system of inverse sequence the vectors of which rotate in an opposite direction from a vector system rotation.

Direct sequence voltages of a stator produce currents, that is, magnetic forces of a direct sequence F_d, but inverse sequence voltages generate inverse currents, that is, magnetic forces of inverse sequence F_i. Magnetic forces F_d, and F, based on the machine size, spread–rotate at the same speed but in the opposite direction. Zero sequence voltages produce zero sequence currents but not the rotating component of the magnetic force and this is the reason why its influence is not taken into consideration in asymmetrical fault analysis [15 – 17].

Magnetic forces F_d and F produce independent fluxes which rotate in opposite directions and in rotor windings indicate EMF whose rotor current response is of a direct and inverse sequence I_{dr} and I_{ir}, and as a result of its interaction with fluxes which produced EMF electromagnetic moment of the

machine appear in a rotor M_d and M_i. The machine effect in asymmetrical regimes can be viewed as a mechanical connection of the two machines on the some axis, the fluxes of which turn in opposite directions.

The scheme presented in Fig. 2 is suitable for the voltage and current system of direct and inverse sequence. If the rotor in respect of movement direction and speed ω (direct sequence components) and rotating field has the sliding s, the speed of the rotor rotation is $\omega_r = \omega(1-s)$. The rotor speed in relation to the rotational field of inverse sequence is:

$$S_i = \frac{-\omega - \omega_r}{\omega} = \frac{-\omega - \omega(1-s)}{\omega} = 2 - s, \quad s_i = 2 - s. \tag{22}$$

(a)

(b)   (c)
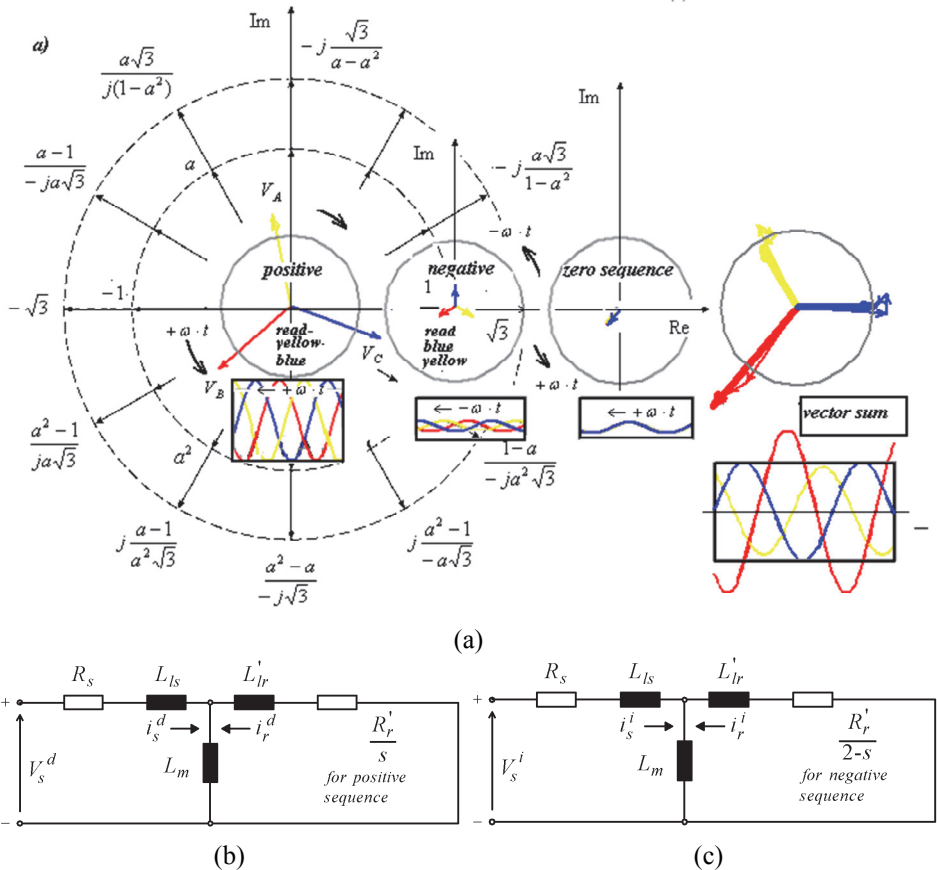
Fig. 2 – *a) Operator value diagram and presentation
of symmetrical components vector rotation;
b) The asynchronous machine scheme, Machine 1 with direct sequence currents,
c) The asynchronous machine scheme, Machine 2 with inverse sequence currents.*

According to Figs. 2a and 2c if reduced rotor resistance is introduced in the equivalent scheme, and if instead of sliding s, $s_i = 2 - s$ is introduced, inverse sequence currents can be determined. According to the symmetrical components method, electromotor force P_{em} is a set of electrical forces determined based on component current values of direct (i_{rd} Machine 1) and inverse (negative) (i_{ri} Machine 2) rotor current:

$$s_i = \frac{-\omega - \omega_r}{\omega} = \frac{-\omega - \omega(1-s)}{\omega} = 2 - s \,, \tag{23}$$

where m_s is the number of stator machine phases.

Total electromagnetic momentum T_{em} in a model creates the fields of direct T_d and inverse sequence T_i:

$$T_{em} = \frac{P_{em}^d}{s\omega^d} - \frac{P_{em}^i}{(2-s)\omega^d} = T^d - T^i, \quad T_{em} = \frac{1}{\omega_d} m_s \left((I_r'^d)^2 \frac{R_2'}{s^2} - \left(I_r'^i \right)^2 \frac{R_2'}{(2-s)^2} \right). \tag{24}$$

# 4 An Adopted Model Analysis and Algorithm Graphic Presentation

The mathematical model defined using equations (12 – 16) and (17 – 24) is non-linear and transient processes can be analysed only by means of a computer. The choice of a coordinate system, the movement equation and transformation to phase variables depends on analysis and available equipment. In Clarke coordinate system (α, β, 0) all variables are sinus and when choosing calculation steps attention is placed on the minimum time constant and adequacy of sinus value approximation. In the case of modelling a system in a coordinate system $d, i, 0$ which is connected to a rotational flux this problem does not exist because within this system (and any system whose rotation is synchronous to the rotational field) all variables in a stationary state appear as constant values [12, 18].

The mathematical model $d, q, 0$ is contained within the `psbpwm` MATLAB Simulink package which is used for transient process analysis in machines. In $\omega t = 0$ case, Clarke transformation matrix is easily derived. The mathematical model of three–phase machine behaviour is described in [10]. Vectors are expressed in a matrix form and equations show coherence between entry variables at asynchronous machine junction with the voltage source.

The computer technology has enabled the development and application of improved mathematical models and algorithms for solving asymmetry problems of electrical machines through simulation [2, 19].

The inverse sequence value system in machines moves at the synchronous speed but in the opposite direction. This fact has given authors an idea to substitute rotation system parameters of the direct sequence value $\omega t = 2\pi \cdot 50$

Gain ($2\pi \cdot 50$) – block pu2radpersec, of the existing model, with different parameter $\omega t \neq -2\pi \cdot 50$ ($-2\pi \cdot 50$) of the some block `pu2radpersec`, Fig. 3, and by so doing test if `psbpwm` Simulink package can be used for analyzing the process with the inverse component present.
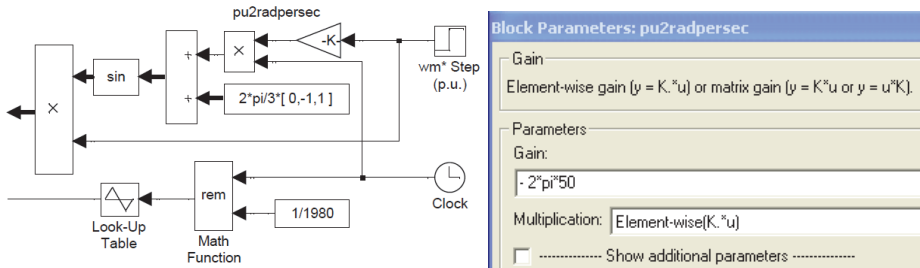
Fig. 3 – *Rotation speed parameter substitution in* `block pu2radpersec` *Parameters Gain* ($-2\pi \cdot 50$).

MATLAB Simulink is mostly used as development environment for designing, implementation and testing of algorithms needed for stationary state analysis (in this case transient state algorithms are analyzed through an adopted scheme for experiment simulation within an existing package `psb` asynchronous machine feed by pwm inverter).

Table 1
Three-phase induction motor "Sever" Subotica testing data [20].

K100, L–4, IP 54, 50[Hz] $V_{AB} = 400$ [V], $p = 2$, $n_{s=d} = 1500\,[\text{min}^{-1}]$					
P_n	n_r	η	$s^d = s = \dfrac{n^d - n_r}{n^d}$	$s^i = s = \dfrac{-n^d - n_r}{-n^d} = (2 - s^d)$	
[kW]	[min^{-1}]	[%]		[min^{-1}]	
2.24	1420	78.7	0.0533	1.94666	
$\cos\varphi$	I_n	M_n	$\dfrac{I_1}{I_n}$	$\dfrac{M_1}{M_n}$	$\dfrac{M_b}{M_n}$	J	m
[%]	[A]	[Nm]				[kgm^2]	[kg]
0.81	5	15	8.2	3.8	4	0.089	35.5

The dynamic state is generated through two models, out of which the first model with an index (*d*) simulates the processes of a single machine which operates in a direct sequence system $\omega_d = 2\pi f$, and the second model with an

index (*i*), simulates the processes and values of an inverse sequence system $\omega_i = -2\pi f$.

Testing–simulation is performed for asymmetrical feeding of a three–phase asynchronous motor "Sever" Subotica the figures of which are given in **Table 1**.

The **Table 2** presents the chosen parameters for measure magnitude current in stator and rotor.

Table 2

The chosen three-phase asynchronous motor parameters
used for simulation in MATLAB sim power system `psbpwm`.

(Magnitude imbalance – positive + negative current) Measure magnitude current in stator and rotor	
Block Parameters AM: Implements a three-phase asynchronous machine	Asynchronous machine 2.24 [kW]
Rotor type	Wound
Nom. Power, $L-L$ voltage and frequency: [P_n [VA], V_n [Rms], f_n [Hz]]:	[2716 220 50]
Stator [R_s [ohm], L_{ls} [H]]:	[0.435 2*2.0e-3]
Rotor [R'_r [ohm], L'_{lr} [H]]:	[0.816 2.0e-3]
Mutual inductance L_m [H]:	69.31e-3

Simulation examples of a transient state in `psbpwm` package, with the existing direct sequence system are shown in Fig. 4a, and Fig. 4b presents an adopted scheme for an inverse sequence system.

The next part that follows presents diagrams which display dependency between electrical and mechanical values of direct and inverse sequence, as to enable comparison of these values in the machine and in order to determine their influence on machine performance. The machine transient state is determined through addition of direct and inverse components of electrical values (voltage and current) according to equation (6) for matrix transformation, equation (23) for electromotor force and equation (24) for electromagnetic moments. Values are then taken from a diagram i_{dr}, i_{ir} to determine electromagnetic moments:

$$T_{em} = \frac{1}{\omega_d} m_s \left(\left(i_r^d\right)^2 \frac{R'_2}{s^2} - \left(i_r^i\right)^2 \frac{R'_2}{\left(2-s\right)^2} \right).$$

(a)

(b)
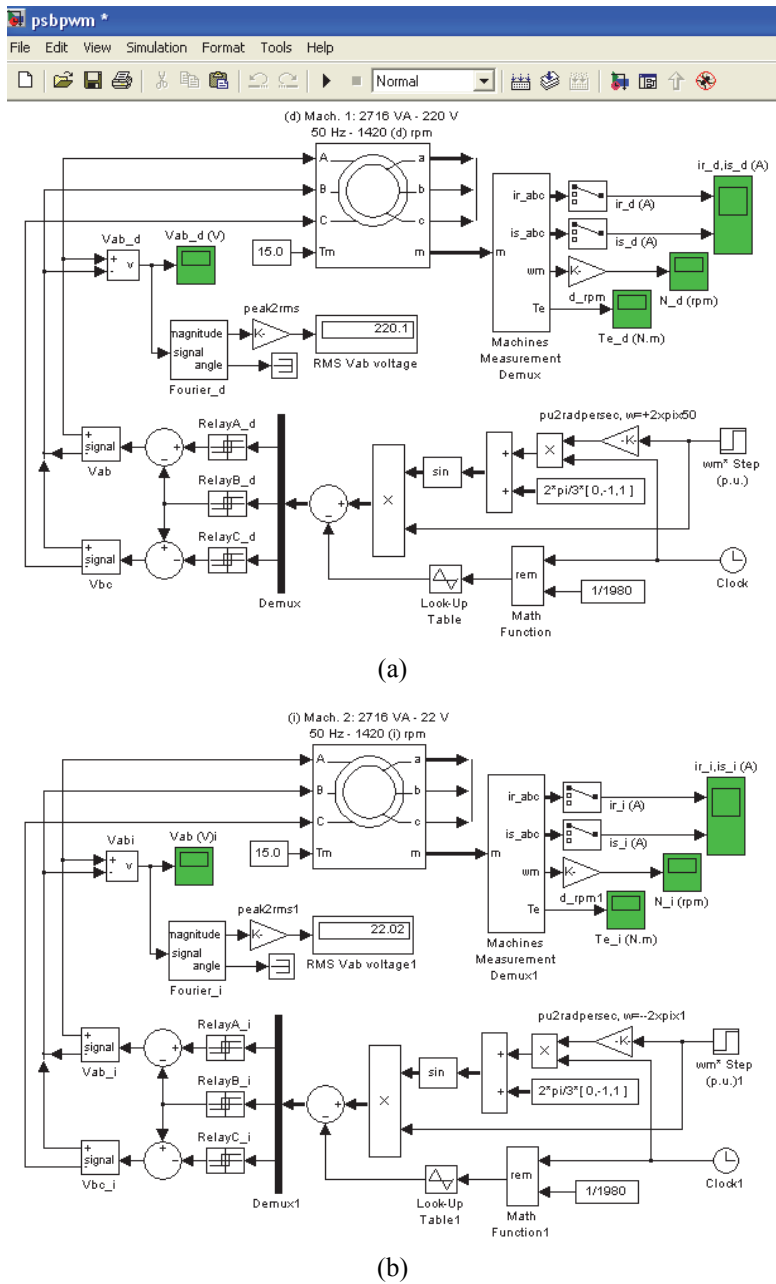
Fig. 4 – *The asynchronous machine performance simulation, a)Machine 1 in a direct sequence system with rotation parameters* $\omega = 2\pi \cdot 50$ *; b) Machine 2 in an inverse sequence system with rotation parameters* $\omega = -2\pi \cdot 50$ *.*
Other parameters can be seen from the scheme.

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5   Simulation Results and Discussion

Stator windings are connected to phase voltage which comes from convertors $V_{AB} = V_{L1} = V_{L2} = 380\,\mathrm{V}$ which coincide with $V_A = V_{L1} = 380 / \sqrt{3} = 219.6\,\mathrm{V}$. Those values are read out from RMS Voltage element of the scheme: direct component voltage V_d and inverse component voltage V_i.

Derived diagrams in relation to the given asymmetry coefficient ($\varepsilon_{ui} = 10$ [%]) of the phase voltage feeding source, that is, in relation to asymmetry conditions presented in Figs. 5 and 6. Fig. 5 contains machine direct components of electrical values: machine phase voltage V_{ABd}, stator currents and reduced rotor currents i_{rd}, i_{sd}, as well as asynchronous machine direct components of mechanical values: machine rotor rotation speed N_d and electromagnetic moment diagram T_{ed}. Fig. 6 also presents diagrams as does Fig. 5, but for electrical and mechanical value inverse components in relation to the given asymmetry coefficient $\varepsilon_{ui} = 10$ [%].

Figure form and values indicate the possibility of attaining component analysis and comparison and machine performance evaluation in the presence of asymmetry. Inverse component values for stator currents and reduced rotor currents, moment and rotation speed of a given asymmetry value confirm that it is possible and justifiable to use the existing algorithm for transient state machine simulation.

Development environment and simulation adapted scheme have enabled simple visualization of the given algorithm, and graphic interface and derived results in a diagram form confirmed and justified its adequacy. It has been proven possible to adopt MATLAB Simulink packages to inverse values and value adequacy of the proposed method for three–phase asynchronous machine asymmetrical process deliberation.

Literature references mostly state that inverse sequence currents create additional heating thus reducing useful mechanical moments and disposable machine strength [6, 21]. Backed by diagram value moment analysis in Figs. 5 and 6, resspectively, a conclusion can be made that feeding asymmetry and asynchronous machine inverse moment components do not influence on machine electromagnetic moment values.

Asymmetrical regime character of three–phase GDS, IEC and national standards are evaluated using asymmetry value coefficient of inverse ε_{ui}, and zero current ε_{u0} [1, 21], and enable inverse current value divergence V_i until 2[%] in relation to network voltage values, $\varepsilon_u^i \leq \varepsilon_{u.doz} = 0.02$:

$$\varepsilon_u^i = V_i / V_n, \quad \varepsilon_u^0 = V_0 / V_n, \tag{25}$$

where V_0, V_i represent voltage symmetrical components of zero and inverse sequence, and V_n represents nominal network voltage.
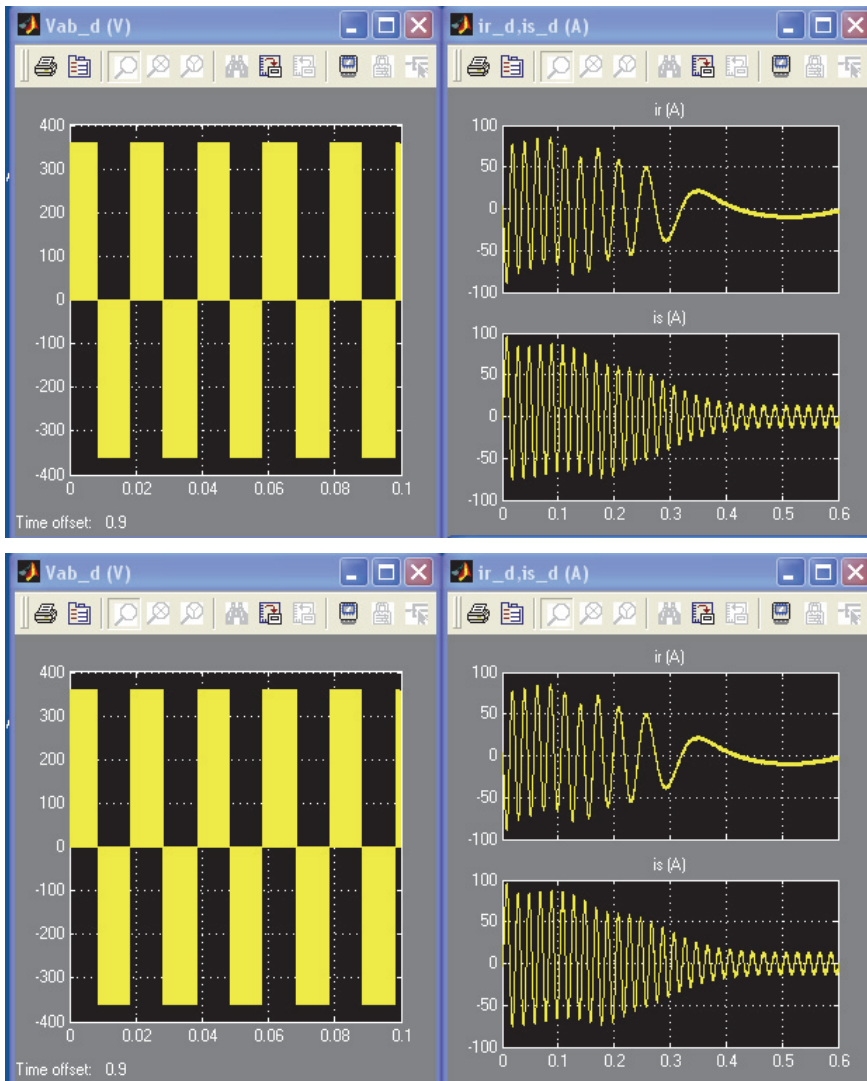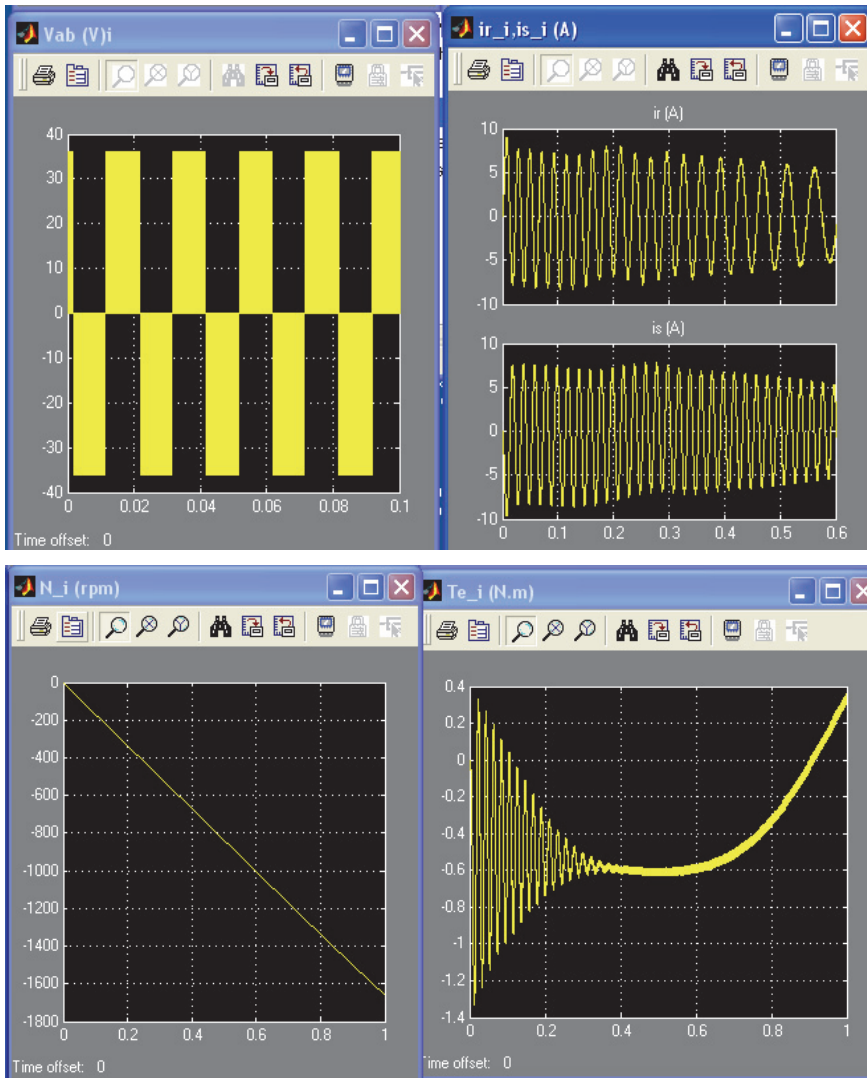
Fig. 5 – *Voltage diagrams, stator currents, rotation and moment speed Machine 1 which operates in a direct sequence system with rotation parameters* $\omega = 2\pi \cdot 50$.

In theory, asymmetrical feeding does not create uneven rotor heating because rotor phase in relation to stator constantly changes [22, 23].

However, appearance of harmonics changes resistance with frequency (rotor current skin effect), and because of a direct field in a rotor, current with frequency, is induced *sf*, and due to inverse field $[(2-s)f]$ frequency values for basic frequency $f = 50\,\text{Hz}$ and sliding $s = 5.33\,[\%]$ are: $sf = 2.666\,\text{Hz}$ and

$[(2-s)f] = 97.333\,\text{Hz}$. This implies that current direct component pre-sence I_d can create increased heating which depends on resistance R_{js}, while inverse currents I_i create even rotor heating $R = C_{skin}R_{js}$. R_{js} represents direct current resistance values of, while C_{skin} is a coefficient which represents skin effect influence and amounts to $C_{skin} = 1.25 - 6$ (cf. [5, 20]).

Fig. 6 – *Voltage diagrams, stator currents, rotation and moment speed Machine 2 which operates in an inverse sequence system with rotation parameters* $\omega = -2\pi \cdot 50$.

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6 Conclusion

Since machine performance extreme regimes are safely simulated as their practical analysis would be difficult and could damage them, computer simulation substitutes the experiment. Therefore, this paper describes a possible adaptation of the asynchronous machine to an existing mathematical model and creation of a new one which presents its behaviour precisely enough, and is at the sometime comprehensive and simple for application. The new algorithm for simulation is made founded on the adapted mathematical model. The advantage of computer simulation is that it is completely safe, for the machine, and its users, and, at the same time, the derived results coincide with the real state and processes.

The asynchronous electric machine derived from a mathematical model can be used for machine performance analysis and operation synthesis, and electrical machine complex operation algorithms can be developed based on this model through vector operation and moment control.

Although a large number of available, ready to use computer programs, which contain complex mathematical models, already exist: VisSim, Mathematica (Wolfram), etc., only MATLAB Simulink program contains ready models for electrical machines of all types.

Ready MATLAB Simulink programs simulate machine performance precisely enough, but independent development of the model, and the program, has special advantages such as detailed insight into all model and program components and change input which would not be possible with other available program packages.

7 Acknowledgments

8 References

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