

Comparative Analysis of Two Gain- and Offset-Compensated Four-Phase Switched-Capacitor Integrators Based on The Second-Order Adams-Bashworth's Integration Method

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Abstract: Gain- and offset- compensated (GOC) modifications of four-phase inverting and noninverting switched-capacitor integrators based on the second-order Adams-Bashworth's integration method are presented. Analytical expressions for the gain, phase and offset voltage errors are derived and compared with the corresponding errors of the earlier ones compensated integrators. The two pairs of GOC integrators are used as building blocks of a bandpass biquad. The performances of the resulting filters are also compared.

Keywords: Filters, Gain- and offset- compensation, Operational amplifiers, Switched-capacitor integrators.

1 Introduction

In designing switched-capacitor (SC) filters various transformations are in use to map the s -plane onto the z -plane. The most popular transformations are the bilinear (BL) and the lossless discrete integrator (LDI), because they preserve the magnitude-frequency response of the continuous-time s -plane prototype filter [1]. In [2 - 6] non-conventional $s \rightarrow z$ transformations based on multistep numerical integration methods were proposed. The second-order Adams-Bashworth's (AB2), the third-order Adams-Moulton's (AM3) and the third-order Milne-Simpson's (MS3) integration algorithms are considered and the corresponding SC integrators are synthesized. These integrators are all derived from the simple inverting analog integrator in which the input resistor is replaced by an SC resistor with topology determined by the integration method and the number of clock phases. Two pairs of four-phase inverting and noninverting AB2 SC integrators were proposed in [5] and [6]. The former uses fewer components, but in the latter a reduction in capacitance spread is achieved. A gain- and offset- compensated (GOC) version of the uncompensated AB2 SC integrators from [5] was proposed in [7].

This paper presents a GOC modification of the four-phase AB2 integrators described in [6]. Analytical expressions for the gain, phase and offset voltage errors are

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derived and compared with the corresponding errors of the GOC integrators from [7]. The two pairs of GOC integrators are used as building block of a bandpass biquad. The performances of the resultant filters are also compared.

2. Circuit architecture and theoretical results

In the following the operational amplifiers (op amps) are assumed to have finite dc gain A_0 and infinite bandwidth. This supposition is adequate for the analysis of SC circuits containing fast and relatively low-gain amplifiers. The nonzero input-referred dc offset voltage of the op amps is modeled as a voltage source at the noninverting input terminal.

Figs. 1, 2 and 3 show the circuits and the clock waveform of the two pairs of four-phase inverting and noninverting AB2 SC integrators, proposed in [5] and [6]. The blocks enclosed in broken line are the inverting (-R) and noninverting (+R) four-phase AB2 SC resistors.

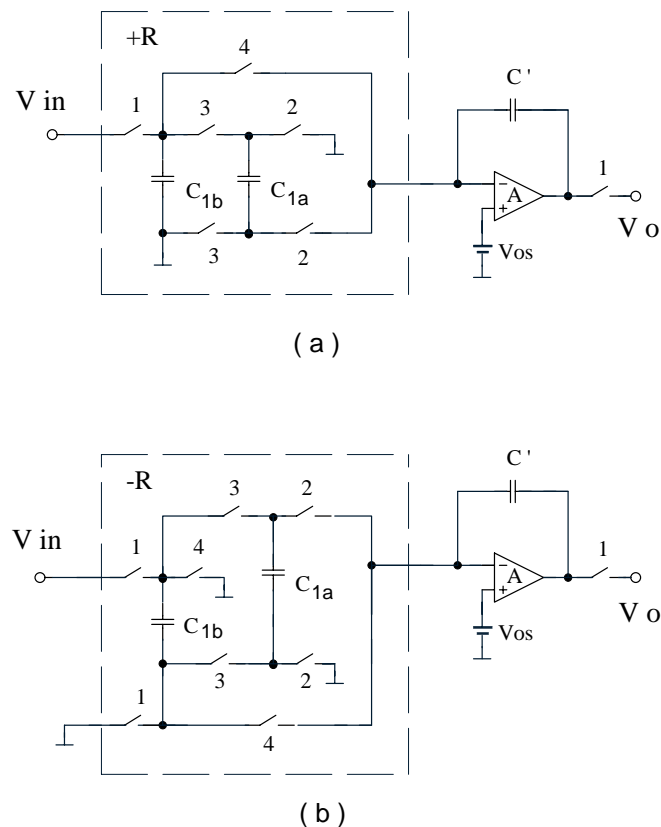
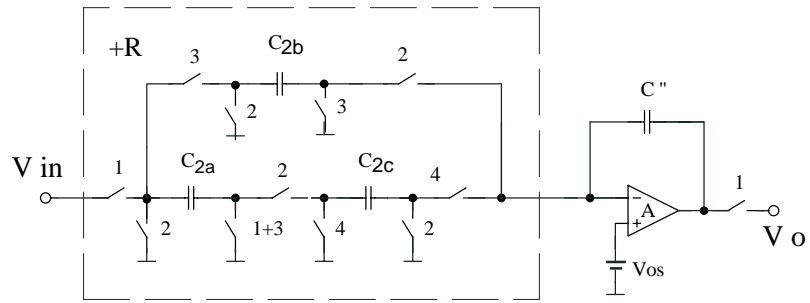
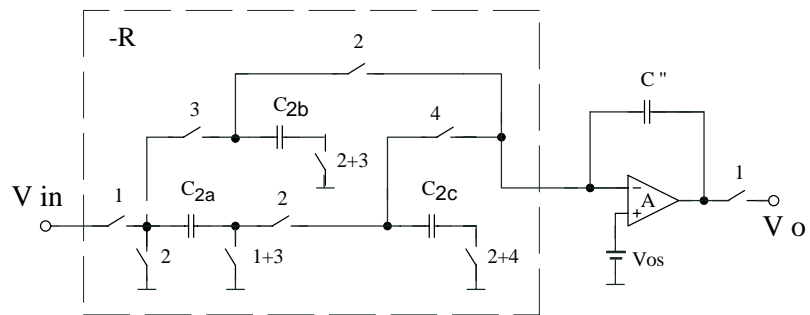


Fig. 1 - Four-phase AB2 SC integrators from [5] (a) inverting integrator (b) noninverting integrator.



(a)



(b)

Fig. 2 - Four-phase AB2 SC integrators from [6] (a) inverting integrator (b) noninverting integrator.

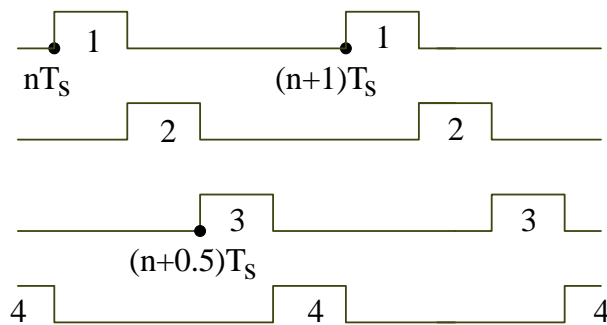


Fig. 3 - Clock waveform.

It is reasonable to assign $C_{1a}=1, C_{1b}=3, C_{2a}=C_{2c}=2, C_{2b}=1, C'=3/4k$ and $C''=1/3k$ units, where $k<1$. Then the z -domain transfer functions of the ideal integrators ($A_0 \rightarrow \infty, V_{os} = 0$) are given by

$$H_{id}(z) = \mp \frac{k(3z^{-1} - z^{-2})}{1 - z^{-1}}. \quad (1)$$

A GOC version of the integrators from Fig. 1 was proposed in [7]. The same GOC technique can be used for the integrators in Fig. 2. The resultant GOC integrators are shown in Fig. 4.

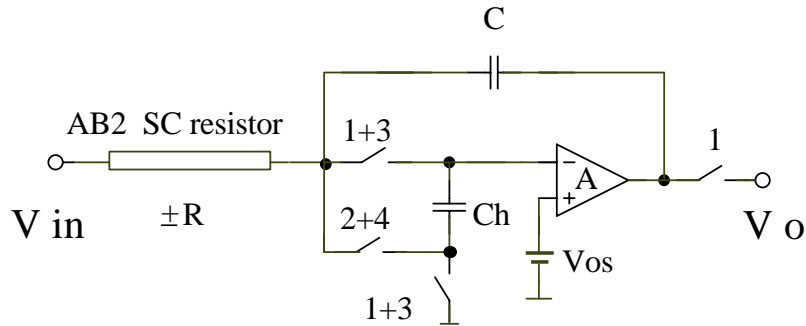


Fig. 4 - GOC four-phase AB2 SC integrators.

The value of the holding capacitor C_h is not critical and can be set equal to the unit capacitance (smallest filter capacitance). Table 1 compares the complexity of the two pairs of GOC AB2 SC integrators in terms of component count and area requirement.

For the SC integrators in Fig. 2 a reduction of 55.56% in capacitance spread is achieved.

The transfer function of a nonideal integrator can be expressed in the continuous time domain as

$$H(j\omega) = H_{id}(j\omega) [1 + m(\omega)] \exp[j\Theta(\omega)]. \quad (2)$$

Table 1
Comparison of the complexity of the two pairs of GOC SC integrators.

GOC modification of the integrators	number of capacitors	number of switches	total C	C spread
in Fig. 1a	4	10	$5+0.75/k$	$0.75/k$
in Fig. 1b	4	12	$5+0.75/k$	$0.75/k$
in Fig. 2a	5	15	$6+1/3k$	$1/3k$
in Fig. 2b	5	13	$6+1/3k$	$1/3k$

Here $m(\omega)$ is the gain error and $\Theta(\omega)$ is the phase error due to the finite amplifier gain. The errors for the GOC version of the integrators from Fig. 1 are given [7] by

$$m_1(\omega) \cong -1 + \left[\left(1 + \frac{1+19k/3}{A_0} \right)^2 + \frac{9k^2}{A_0^4 \tan^2(0.5\omega T_s)} \right]^{-\frac{1}{2}} \quad (3)$$

and

$$\Theta_1(\omega) \cong \frac{3k}{A_0^2 \tan(0.5\omega T_s)} + \frac{k \sin \omega T_s}{A_0} \left(\frac{4}{5-3\cos \omega T_s} - \frac{1}{3} \right). \quad (4)$$

The corresponding expressions for the GOC modification of the integrators from Fig. 2 are

$$m_2(\omega) \cong -1 + \left[\left(1 + \frac{1+8k}{A_0} \right)^2 + \frac{9k^2}{A_0^4 \tan^2(0.5\omega T_s)} \right]^{-\frac{1}{2}} \quad (5)$$

and

$$\Theta_2(\omega) \cong \frac{3k}{A_0^2 \tan(0.5\omega T_s)} + \frac{k \sin \omega T_s}{A_0} \left(\frac{9}{5-3\cos \omega T_s} - 4 \right). \quad (6)$$

The two pairs of GOC integrators have nearly frequency independent and comparable gain errors. The errors $m_2(\omega)$ are somewhat larger. The phase error $\Theta_1(\omega)$ is always positive, while the error $\Theta_2(\omega)$ crosses through zero at signal frequency

$$f_0 = \frac{f_s}{2\pi} \arccos \left\{ \frac{1}{24} \left[23 - \frac{9}{A_0} - \sqrt{1 + \frac{9}{A_0} \left(34 + \frac{9}{A_0} \right)} \right] \right\} \quad (7)$$

and then increases in negative values.

To compare the two pairs of GOC integrators in terms of offset voltage errors it is advantageous to use the known $\alpha \beta \gamma$ representation defined in [8]. The derived expressions for the dc gain $H(0)$ and for the offset suppression factor γ are summarized in Table 2.

Table 2
Comparison of GOC SC integrators in terms of dc gain $H(0)$ and suppression factor γ .

GOC modification of the integrators	DC gain $H(0)$	Suppression factor γ
in Fig. 1	$\mp A_0^2 \frac{1 + \frac{3+4k/3}{A_0}}{3}$	$\frac{6k}{A_0} \left(1 - \frac{2+16k/3}{A_0} \right)$
in Fig. 2	$\mp A_0^2 \frac{1 + \frac{3(1+k)}{A_0}}{3}$	$\frac{6k}{A_0} \left(1 - \frac{2+10k}{A_0} \right)$

It is seen that the two pairs of GOC integrators have nearly the same dc gains and offset voltage errors.

3 Bandpass biquad with different integrators

As an example providing a comparison of the integrators considered, a bandpass biquad was designed. The ideal filter specifications are: pole quality factor $Q_p = 0.9969395$, pole frequency $f_p = 1000.04\text{Hz}$, peak gain $H_p = 1.000059$, sampling frequency $f_s = 128\text{kHz}$. The corresponding SC design with the uncompensated integrators from Figs.1 and 2 is shown in Fig. 5.

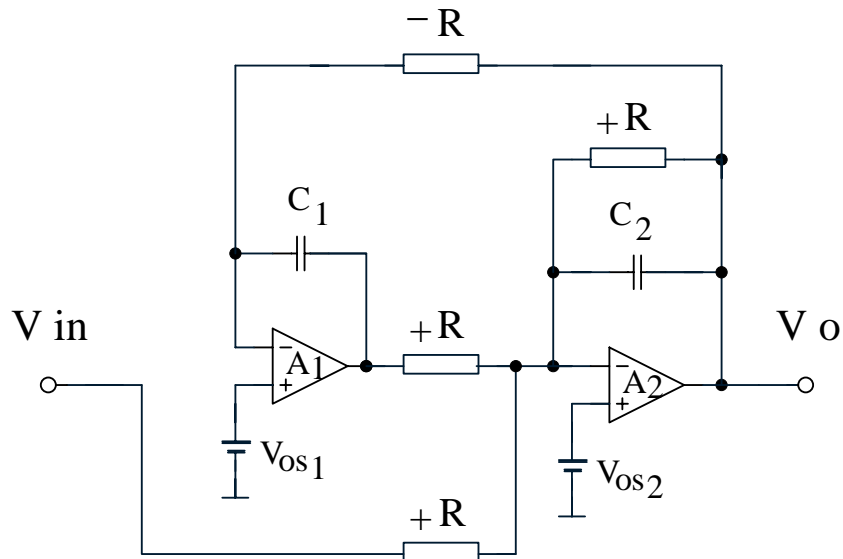


Fig. 5 - Bandpass SC biquad with uncompensated AB2 SC integrators.

The relative capacitors values C_1 and C_2 are: $C_1' = C_2' = 30.588413$ - for the biquad with the integrators in Fig. 1; $C_1' = C_2' = 13.59485$ - for the biquad with the integrators in Fig. 2. Subsequently, the integrators in the uncompensated structures (Fig. 5) have been replaced by the GOC integrators from Fig. 3, where $Ch_1 = Ch_2 = 1$ was chosen. Figs. 6 and 7 show the simulated magnitude responses of the bandpass biquad designed with the uncompensated and with the GOC AB2 SC integrators for $A_{01} = A_{02} = A_0 = 100$. The corresponding curves of the GOC filters practically converge on the scale chosen to those obtained by initially assuming the op amps to be ideal ($A_0 \rightarrow \infty$). It is observed that the finite-gain effect of the op amps is reduced for the filters with the GOC integrators.

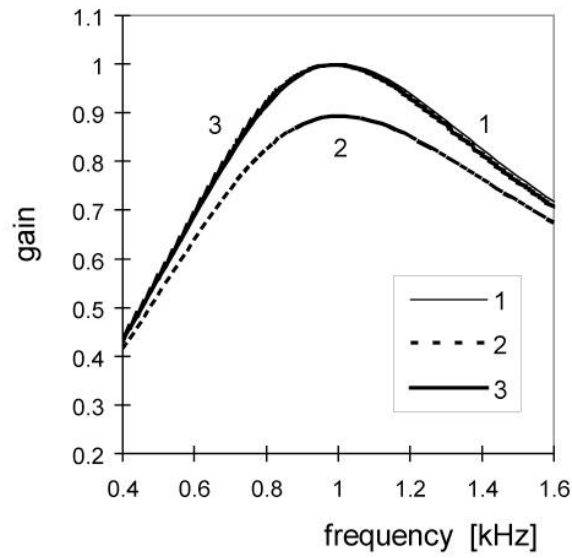


Fig. 6 - Simulated magnitude responses of bandpass biquad with the integrators in Figs. 1 and 3,
 1 - ideal response ($A_0 \rightarrow \infty$) 2 - with the non-GOC integrators ($A_0 = 100$)
 3 - with the GOC integrators ($A_0 = 100$).

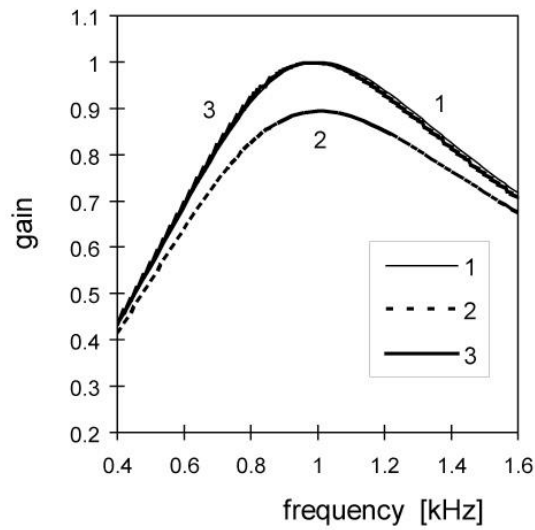


Fig. 7 - Simulated magnitude responses of bandpass biquad with the integrators in Figs. 2 and 3,
 1 - ideal response ($A_0 \rightarrow \infty$) 2 - with the non-GOC integrators ($A_0 = 100$)
 3 - with the GOC integrators ($A_0 = 100$).

Table 3 summarizes the actual pole frequencies f_{pa} , the actual pole quality factors Q_{pa} , the gains $H_a(f_{pa})$ at the pole frequencies f_{pa} , and the relative errors δf_{pa} , δQ_{pa} and $\delta H_a(f_{pa})$ of the filter with the two pairs of GOC SC integrators for finite op amp gain $A_0 = 100$. It is seen that the two GOC filter structures exhibit nearly the same relative errors δf_{pa} , δQ_{pa} and $\delta H_a(f_{pa})$. It is due to the nearly the same gain and phase errors in the vicinity of the ratio f_p / f_s .

Table 3
Performance parameters of the bandpass biquad with the GOC integrators
for op amp gain $A_0 = 100$.

GOC modification of the integrators	f_{pa} (Hz)	δf_{pa} (%)	Q_{pa}	δQ_{pa} (%)	$H_a(f_{pa})$	$\delta H_a(f_{pa})$ (%)
in Fig. 1	987.54	-1.250	0.99711	0.017	0.99879	-0.1269
in Fig. 2	987.16	-1.288	0.99717	0.023	0.99885	-0.1209

The pole frequency shifts can be centered around $f_p = 1000$ Hz by modifying the capacitor values $C_1 = C_2$ according to the relations

$$C_1' = C_2' = C_M' = \frac{3f_s}{4\pi f_a}, \quad C_1'' = C_2'' = \frac{f_s}{3\pi f_a}, \quad (8)$$

where

$$f_a = \frac{f_s}{2\pi} \frac{1 + 3 \tan^2(\pi f_p' / f_s)}{1 + 4 \tan^2(\pi f_p' / f_s)} \sin(2\pi f_p' / f_s) \quad (9)$$

for $f_p' = 2000 - f_{pa}$.

One obtains $C_M' = 30.212732$ and $C_M'' = 13.422853$.

The additional compensation is effective for op amps with low but precisely known and stable dc gain. The corresponding performance parameters of the two GOC bandpass biquads with gain variation $\pm 8\%$ are summarized in Table 4.

Table 4
Performance parameters of the GOC bandpass biquads with additional compensation.

A_0	δf_{pa} C_M'	(%) C_M''	δQ_{pa} C_M'	(%) C_M''	$\delta H_a(f_{pa})$ C_M'	(%) C_M''
92	-0.13	-0.136	0.0036	0.0076	-0.1487	-0.1427
100	-0.023	-0.025	0.012	0.017	-0.1266	-0.1209
108	0.069	0.07	0.018	0.022	-0.1091	-0.1040

The steady state output voltages of the bandpass biquads designed with the GOC versions of the two pairs of SC integrators from Figs. 1 and 2 for $A_0 = 100$, are respectively

$$\lim_{n \rightarrow \infty} Vo(n) = -0.02970Vos_1 + 2.649 \cdot 10^{-5} Vos_2, \quad (10)$$

$$\lim_{n \rightarrow \infty} Vo(n) = -0.02971Vos_1 + 2.651 \cdot 10^{-5} Vos_2. \quad (11)$$

For the corresponding uncompensated filters we have

$$\lim_{n \rightarrow \infty} Vo(n) = -2.9041Vos_1 + 0.2611Vos_2, \quad (12)$$

$$\lim_{n \rightarrow \infty} Vo(n) = -2.9037Vos_1 + 0.2609Vos_2. \quad (13)$$

It is seen that the influence of Vos is also reduced for the GOC bandpass biquads.

4 Conclusion

Gain- and offset- compensated four-phase inverting and noninverting switched-capacitor integrators based on the second-order Adams-Bashworth's integration method have been presented and compared with previously proposed compensated integrators. For the novel integrators a reduction of 55.56 % in capacitance spread is achieved. The two pairs of compensated integrators have been used as building blocks of a bandpass biquad. The resultant filters exhibit nearly the same relative errors in the pole frequency f_p , in the pole Q-factor Q_p and in the amplitude H_p at the pole frequency. It is due to the nearly the same gain and phase errors of the integrators considered. The pole frequency shifts have been centered around the ideal pole frequency by modifying two capacitances.

5 References

- [1] H. M. Yassine: General analogue-to-digital transformation, IEE Proceedings, Vol. 133, Pt.G., No2, pp. 89 - 94, April 1986.
- [2] V. Georgiev, T. Stoyanova: Two-phase switched-capacitor integrator based on the second-order Adams-Bashworth's integration method, Proceedings of TU-Rousse, Bulgaria, Vol. 26, pp.63 - 69, 1984 (in Bulgarian).
- [3] V. Georgiev, T. Stoyanova: Design of switched-capacitor circuits based on higher-order z-transforms, Proceedings of the International conference Microelectronics'86, 23 - 25 October, Plovdiv, Bulgaria, Vol.3, pp.145 - 157, 1986 (in Russian).
- [4] S. Farchy, V. Georgiev, T. Stoyanova: SC-resistor based on multistep numerical integration, Proc. of the Int. Conf. " MIKROELEKTRONIKA'84 ", Praha, 2-6 Sept., Vol. 2, pp. 102 - 103, 1984.

Nikolay Radev, Kantcho Ivanov

- [5] S. Farchy, V. Georgiev, T. Stoyanova: SC-Widerstands-modelle auf der Grundlage der rechnergestützten Integration höherer Ordnung, 29 Int. Wiss. Kol. Ilmenau, 26 - 30 Oct., Postreihe A1 (p. 22), 1984.
- [6] V. Georgiev: Contributions in the analysis and the synthesis of some switched-capacitor circuits (SC-circuit), Dissertation for obtaining the D. Sc. degree, Sofia, Bulgaria, 1995 (in Bulgarian).
- [7] N. Radev, K. Ivanov: Four-phase switched-capacitor second-order Adams-Bashworth integrators with gain and offset compensation, *Elektrotechnica & Elrctronica, Bulgaria*, Vol. 38, No 3 - 4, pp.14 - 19, 2003.
- [8] W. H. Ki, G. C. Temes: Gain- and offset- compensated switched-capacitor filters, in *Proc. IEEE ISCAS'91*, pp. 2829 - 2832, 1991.