SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 2, No. 2, November 2005, 163 - 172

The Point Ground Electrode in Vicinity of the Semi-Spherical Inhomogenity

Nenad N. Cvetković[1], Predrag D. Rančić[2]

Abstract: Characterisation of the point ground electrode placed in the surroundings or inside of the semi-spherical earth inhomogenity and fed by low frequency (LF) current using isolated earthing conductor, is presented in this paper. The ground impedance (resistance and reactance) and potential distribution on the ground surface are determined. Image theory for two-layer semi-conducting media, as well as for the one point electrode placed nearby or inside of the spherical body is used during the analysis.

Keywords: Grounding system, Impedance, Potential, Green's function.

1 Introduction

The spherical and semi-spherical inhomogenity in the surroundings of the grounding can be presented in analysis and solving of different problems. For example, concrete foundation of the pillar can be approximately treated as inhomogenity of this kind. Also, the obtained results can be used for characterisation (determining impedance, step voltage, touch voltage and potential distribution) of the linear grounding electrode placed at the arbitrary direction in the surroundings or inside of analyzed inhomogenity, treating earth as a semi-conducting medium. Combination of the quasistationary image theory for two-layer semi-conducting media and for semi-conducting spherical inhomogenity, was applied. The result of the previously described analysis is the Green's function of the point electrode placed nearby or inside of the semi-spherical earth inhomogenity [1, 2].

2 Theoretical Background

The point ground electrode having radius r_0 fed by LF current I and placed inside and outside of the semi-spherical semi-conducting inhomogenity of radius a is observed (Fig. 1a-b). The electrode is placed at the depth h. The earth is also treated as semi-conducting medium. The normal distance between the

[1]Faculty of Electronic Eng. of Nis, A. Medvedeva 14, 18000 Nis, S&M, E-mail: csone@elfak.ni.ac.yu
[2]Faculty of Electronic Eng. of Nis, A. Medvedeva 14, 18000 Nis, S&M, E-mail: prancic@elfak.ni.ac.yu

electrode and the center of the semi-spherical inhomogenity is L. The center of the spherical coordinate system coincides with the center of semi-spherical inhomogenity. The position of the point electrode is defined by the values of the spherical coordinates $d = (h^2 + L^2)^{1/2}$ and $\alpha = \arctan(L/h)$.

The conductivities, permitivitties and permeabilities of the system shown in Fig. 1a-b are labelled with σ_i, $\varepsilon_i = \varepsilon_0\varepsilon_{ri}, \mu_i = \mu_0, i = 0,1,2$, where indexes 0, 1 and 2 correspond to the free space, semi-conducting ground and semi-spherical inhomogenity, respectively. The complex conductivities are: $\underline{\sigma}_0 = \mathrm{j}\omega\varepsilon_0$ for free space ($\sigma_0 = 0$), $\underline{\sigma}_1 = \sigma_1 + \mathrm{j}\omega\varepsilon_1$ for semi-conducting ground and $\underline{\sigma}_2 = \sigma_2 + \mathrm{j}\omega\varepsilon_2$ for semi-spherical inhomogenity, where ω is the angular frequency.

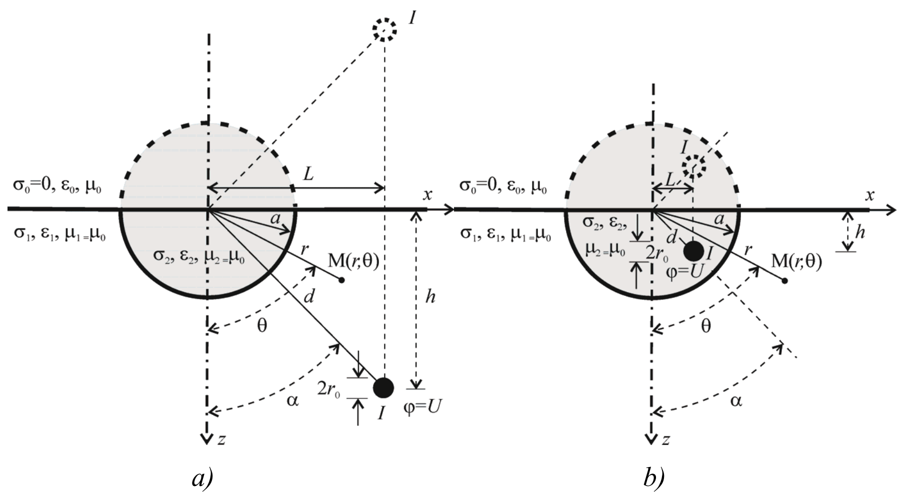


Fig. 1 – *Point ground electrode outside (a) and inside (b) of the semi-spherical inhomogenity.*

2.1 Determination of the potential and grounding impedance

Procedure of potential determination of the system shown in Fig. 1a-b is based on combine using of the quasistationary image theory for two-layer semi-conducting media [1] and image representation of the point electrode placed near the semi-conducting spherical body [2]. In general case, the equivalent system can be formed combining application of image theorem in plane semi-conductive mirror and semi-conductive semi-spherical inhomogenity. In obtained case, the condition $\underline{\sigma}_1, \underline{\sigma}_2 >> \underline{\sigma}_0$ is satisfied. After applying the image theory, the equivalent system is formed of point electrode, and its image placed at the distance h above the boundary surface, at normal distance L from the center of the spherical coordinate system. The equivalent image current is $R_{z10}I$ (when grounding electrode is inside of the inhomogenity) or $R_{z20}I$ (when source is outside of the inhomogenity), where

$$R_{z10} = (\underline{\sigma}_1 - \underline{\sigma}_0)/(\underline{\sigma}_1 + \underline{\sigma}_0) \text{ and } R_{z20} = (\underline{\sigma}_2 - \underline{\sigma}_0)/(\underline{\sigma}_2 + \underline{\sigma}_0)$$

are the reflection coefficients. By that, since it is $\underline{\sigma}_1, \underline{\sigma}_2 >> \underline{\sigma}_0$, it can be assumed that $R_{z10}, R_{z20} \approx 1$. Also, in this case, after applying the image theory, the semi-spherical inhomogenity can be treated as a spherical semi-conducting body. The potential of the field point in the ground is obtained as a result of the potential superposition of the source and its images. This potential presents the Green's function for the point electrode placed nearby or inside of the semi-spherical earth inhomogenity (Fig. 1a-b), when $\underline{\sigma}_1, \underline{\sigma}_2 >> \underline{\sigma}_0$. It is assumed that source and point in which potential is determined are placed in the same $\psi = \mathrm{c}^{\mathrm{te}}$ -plane of thespherical coordinate system. Now, potential can be expressed as

$$\varphi = \begin{cases} \dfrac{I}{4\pi\underline{\sigma}_1}\left\{\dfrac{1}{R_1} + R_{z12}\dfrac{a}{d}\left[\left(\dfrac{1}{R_2} - \dfrac{1}{r}\right) + \dfrac{T_{z12}}{2a}\ln\left(\dfrac{r - D\cos(\theta-\alpha) + R_2}{2r}\right)\right] + \right. \\ \left. + \left[\dfrac{1}{R_{1l}} + R_{z12}\dfrac{a}{d}\left(\left(\dfrac{1}{R_{2l}} - \dfrac{1}{r}\right) + \dfrac{T_{z12}}{2a}\ln\left(\dfrac{r + D\cos(\theta+\alpha) + R_{2l}}{2r}\right)\right)\right]\right\}, & r \geq a \\ \dfrac{I}{4\pi\underline{\sigma}_1}\left\{\dfrac{T_{z12}}{R_1} - R_{z12}\dfrac{1}{d}\left[1 - \dfrac{T_{z12}}{2}\ln\left(\dfrac{d - r\cos(\theta-\alpha) + R_1}{2d}\right)\right] + \right. \\ \left. + \left[\dfrac{T_{z12}}{R_{1l}} - R_{z12}\dfrac{1}{d}\left(1 - \dfrac{T_{z12}}{2}\ln\left(\dfrac{d + r\cos(\theta+\alpha) + R_{1l}}{2d}\right)\right)\right]\right\}, & r \leq a \end{cases} \tag{1a}$$

when electrode is placed outside, and

$$\varphi = \begin{cases} \dfrac{I}{4\pi\underline{\sigma}_1}\left\{\dfrac{T_{z12}}{R_1} - R_{z12}\dfrac{1}{r}\left[1 - \dfrac{r}{a}\dfrac{T_{z12}}{2}\ln\left(\dfrac{r - d\cos(\theta-\alpha) + R_1}{2r}\right)\right] + \right. \\ \left. + \left[\dfrac{T_{z12}}{R_{1l}} - R_{z12}\dfrac{1}{r}\left(1 - \dfrac{r}{a}\dfrac{T_{z12}}{2}\ln\left(\dfrac{r + d\cos(\theta+\alpha) + R_{1l}}{2r}\right)\right)\right]\right\}, & r \geq a \\ \dfrac{I}{4\pi\underline{\sigma}_1}\left\{\dfrac{\underline{\sigma}_1}{\underline{\sigma}_2}\dfrac{1}{R_1} - R_{z12}\dfrac{a}{d}\left[\left(\dfrac{\underline{\sigma}_1}{\underline{\sigma}_2}\dfrac{1}{R_2} + \dfrac{d}{a^2}\right) - \dfrac{d}{a^2}\dfrac{T_{z12}}{2}\ln\left(\dfrac{D - r\cos(\theta-\alpha) + R_2}{2D}\right)\right] + \right. \\ \left. + \left[\dfrac{\underline{\sigma}_1}{\underline{\sigma}_2}\dfrac{1}{R_{l1}} - R_{z12}\dfrac{a}{d}\left(\left(\dfrac{\underline{\sigma}_1}{\underline{\sigma}_2}\dfrac{1}{R_{2l}} + \dfrac{d}{a^2}\right) - \dfrac{d}{a^2}\dfrac{T_{z12}}{2}\ln\left(\dfrac{D + r\cos(\theta+\alpha) + R_{2l}}{2D}\right)\right)\right]\right\}, & r \leq a \end{cases} \tag{1b}$$

when electrode is placed inside of the semi-spherical inhomogenity. By that are:

$$R_1 = [r^2 + d^2 - 2rd\cos(\alpha - \theta)]^{1/2},$$

$$R_2 = [r^2 + D^2 - 2rD\cos(\alpha - \theta)]^{1/2},$$

$$R_{1I} = [r^2 + d^2 + 2rd\cos(\alpha + \theta)]^{1/2}$$

and

$$R_{2I} = [r^2 + D^2 + 2rD\cos(\alpha + \theta)]^{1/2}.$$

The reflection and transmission coefficients R_{z12} and T_{z12} in expressions (1a-b) are as follows:

$$R_{z12} = T_{z12} - 1 = (\underline{\sigma}_1 - \underline{\sigma}_2)/(\underline{\sigma}_1 + \underline{\sigma}_2).$$

The distance of the images from the center of the semi-conducting sphere, obtained using the image theory for the spherical semi-conducting body, is labelled with $D = a^2/d$.

In the frame of the presented analysis, it is assumed that LF domain is considered ($f = 50\,\text{Hz}, \omega = 100\pi\,\text{rad/s}$). After determining the potential on the electrode surface $\varphi = U$, it is possible to obtain the ground electrode impedance that has capacitive character, $\underline{Z}_g = U/I = R_g - \mathrm{j}X_g$.

3 Numerical Results

The resistance and reactance of the point grounding ground electrode having radius r_0 and fed by LF current I shown in Fig. 1a (when point grounding is placed outside of inhomogenity), versus the distance L, for different σ_1/σ_2 ratio as a parameter, are shown in Fig. 2. The values of the parameters are $h = 0.5\,\text{m}$, $a = 1\,\text{m}$, $r_0 = 2.5\,\text{cm}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 3$ and $\sigma_1 = 0.01\,\text{S/m}$.

The potential distribution on the ground surface for point grounding shown in Fig. 1a, for different σ_1/σ_2 ratio as a parameter, is shown in Fig. 3. The parameters' values are $L = 2\,\text{m}$, $h = 1\,\text{m}$, $a = 1\,\text{m}$, $r_0 = 2.5\,\text{cm}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 3$ and $\sigma_1 = 0.01\,\text{S/m}$.

The resistance and reactance of the point grounding from Fig. 1b (when point grounding is placed inside of the inhomogenity), versus the distance L, for different σ_1/σ_2 ratio as a parameter, are shown in Fig. 4. The values of the parameters are $h = 0.5\,\text{m}$, $a = 1\,\text{m}$, $r_0 = 2.5\,\text{cm}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 3$ and $\sigma_1 = 0.01\,\text{S/m}$.

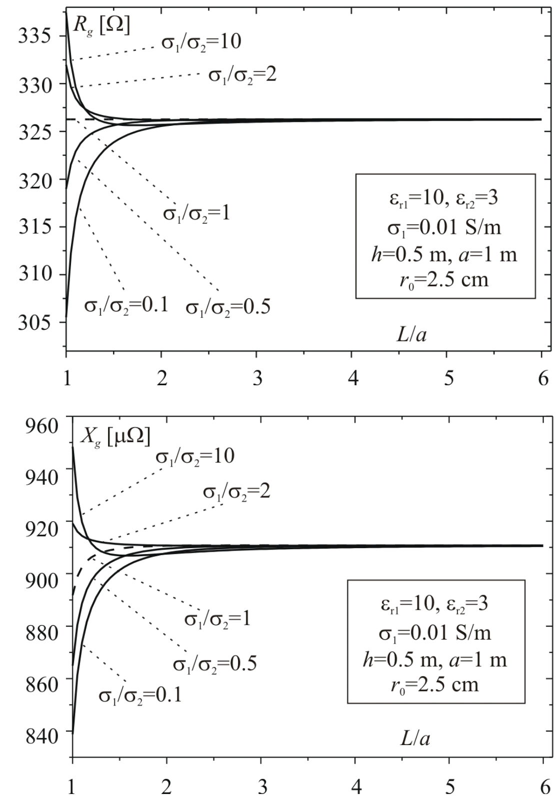


Fig. 2 – *Resistance and reactance of the point ground electrode (Fig. 1a).*

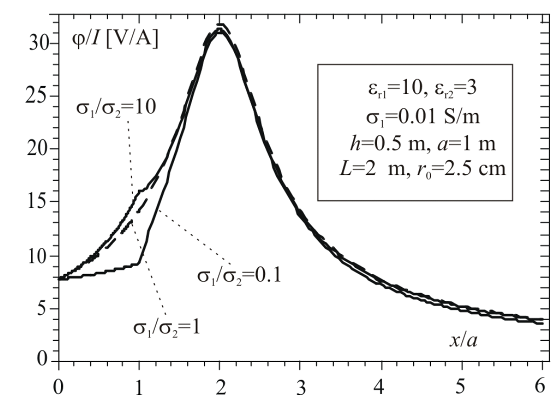


Fig. 3 – *Normalized potential distribution on the ground surface (Fig. 1a).*

The potential distribution on the ground surface for point grounding shown in Fig. 1b, for different σ_1/σ_2 ratio as a parameter, is shown in Fig. 5. The parameters' values are $L = 0.5\,\mathrm{m}$, $h = 0.3\,\mathrm{m}$, $a = 1\,\mathrm{m}$, $r_0 = 2.5\,\mathrm{cm}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 3$ and $\sigma_1 = 0.01\,\mathrm{S/m}$.

The resistance and reactance of the point grounding placed at the surface ($h = 0$) versus the distance L, for different σ_1/σ_2 ratio as a parameter, are presented in Fig. 6 (when point grounding is placed on the inhomogenity surface), and Fig. 7a-b (when grounding is on the earth surface). The values of the parameters are $a = 1\,\mathrm{m}$, $r_0 = 2.5\,\mathrm{cm}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 3$ and $\sigma_1 = 0.01\,\mathrm{S/m}$.

When the point grounding is placed in the center of the inhomogenity, it is possible to obtain its complex impedance in the closed form as

$$\underline{Z}_g = R_g - \mathrm{j}X_g = \frac{1}{2\pi\underline{\sigma}_1 r_0}\left[\frac{r_0}{a}\left(1-\frac{\underline{\sigma}_1}{\underline{\sigma}_2}\right)+\frac{\underline{\sigma}_1}{\underline{\sigma}_2}\right]. \tag{2}$$

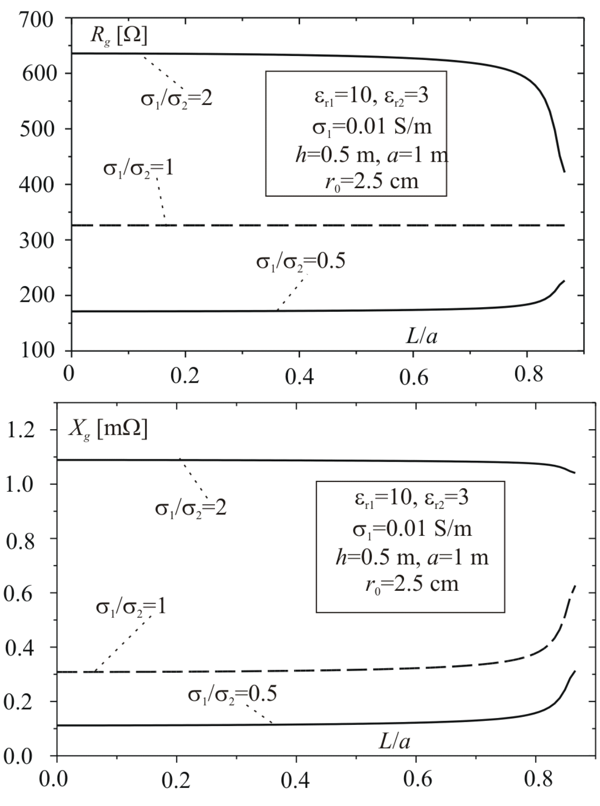


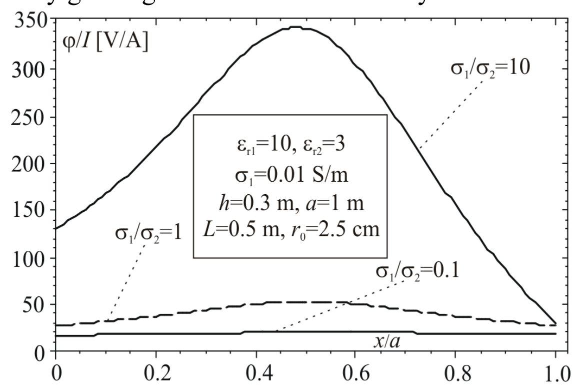
Fig. 4 – *Resistance and reactance of the point ground electrode (Fig. 1b).*

Also in this case of $h=0$, when the point grounding is placed at large distance from inhomogenity ($L >>> a$), with increasing L, the complex impedance approaches the limit value

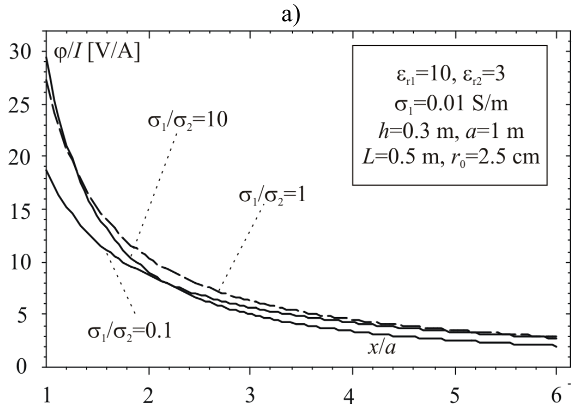
$$\underline{Z}_g = R_g - \mathrm{j} X_g = \frac{1}{2\pi \underline{\sigma}_1 r_0} \cdot \tag{3}$$

The comparison of resistance obtained using expressions (1b) and (2), when point electrode is in the center of the inhomogenity, is presented in **Table 1**. The values of the parameters are $a = 1\,\mathrm{m}$, $r_0 = 2.5\,\mathrm{cm}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 3$ and $\sigma_1 = 0.01\,\mathrm{S/m}$. Good agreement between compared results is notable.

The compared results for resistance obtained using expressions (1a) and (3), (the point electrode is at large distance from inhomogenity) are given in **Table 2**. The values of the parameters are $a = 1\,\mathrm{m}$, $r_0 = 2.5\,\mathrm{cm}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 3$ and $\sigma_1 = 0.01\,\mathrm{S/m}$. Very good agreement can be noticed yet for $L = 1.5a$.



a)



b)

Fig. 5 – *Normalized potential distribution on the inhomogenity surface (a), and on the earth surface (b) for the grounding electrode shown in Fig. 1b.*

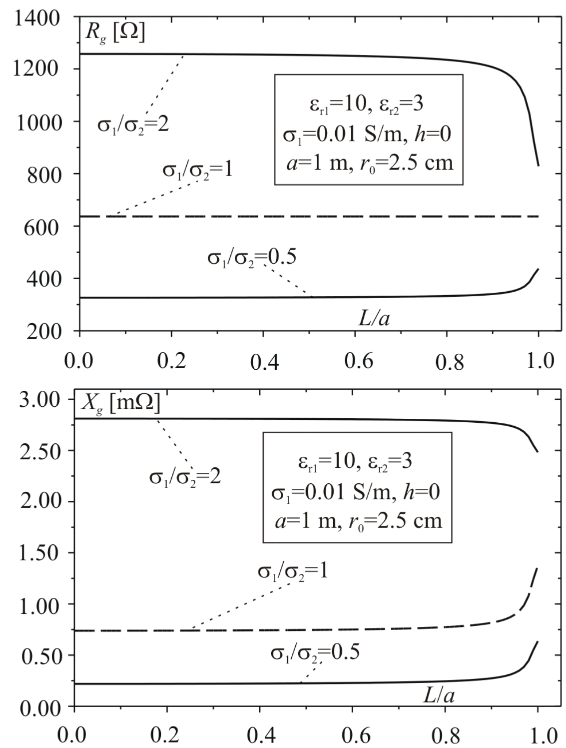


Fig. 6 – *Resistance and reactance of the point ground electrode when electrode is placed on the inhomogenity surface (Fig. 1b).*

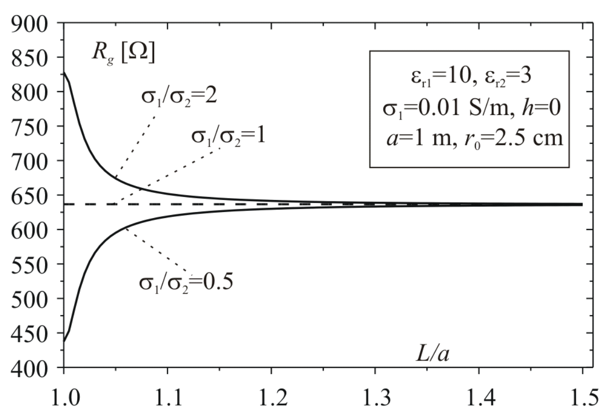


Fig. 7a – *Resistance of the point ground electrode when electrode is placed on the earth surface (Fig. 1a).*

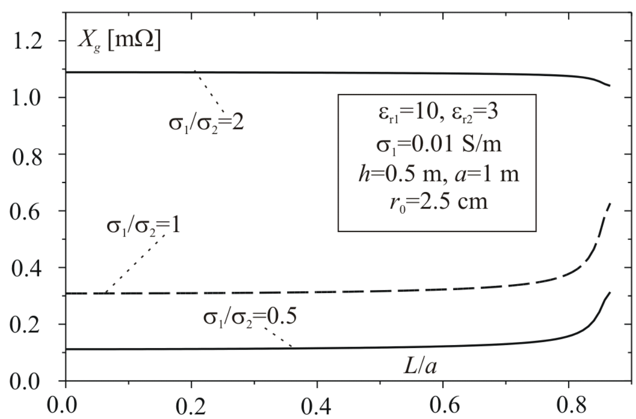


Fig. 7b – *Reactance of the point ground electrode when electrode is placed on the earth surface (Fig. 1a).*

Table 1

Resistance of the point electrode $R_g\,(\Omega)$ placed in the inhomogenity center and value obtained using expr. (2).

	σ_1/σ_2			
	0.1	0.5	2	10
appr. value	77.985921545721	326.26764063081	1257.324078523	6222.9579797
expr. (2)	77.985921545723	326.26764063086	1257.324078527	6222.9579800

Table 2

Resistance of the point electrode $R_g\,(\Omega)$ placed at $L = 1.5a$ from the inhomogenity center, and value obtained using exp. (3).

	σ_1/σ_2			
	0.1	0.5	2	10
appr. value	632.457420	635.427795	637.119070	636.608732
expr. (3)	636.619772			

4 Conclusion

The potential distribution on the ground surface in the vicinity of the point ground electrode placed inside and outside of spherical inhomogenity and fed by LF current using the isolated earthing conductor, as well as point electrode complex impedance, are determined in this paper. The content of this paper presents introductory analysis for solving more complex problems such as characterisation of the linear ground electrode placed in arbitrary direction in the vicinity of the semi-conducting inhomogenity or inside of the inhomogenity treating ground as a semi-conducting medium.

5 Acknowledgement

Authors dedicate the paper to the late professor Dragutin M. Veličkovic (1942-2004).

First author thanks the DAAD foundation and the Technical University of Ilmenau, Germany for the significant support of different kind in the framework of the Joint Project.

6 References

[1] P. D. Rancic: A new concept for linear grounding systems analysis (invited paper), Fourth International Symposium of Applied Electrostatics, PES 96, Nis, Yugoslavia, 22 – 24. May, 1996, Proceedings of Papers, pp. 103-116.

[2] D. M. Velickovic: Green's function of spherical body, Euro Electromagnetics, EUROEM'94, 30. May – 4. Juni, 1994, Bordeaux, France, Conference Proceedings, THp-09-04.

[3] P. D. Rancic, L. V. Stefanovic, Dj. R. Djordjevic: A New Model of the Vertical Ground Rod in Two-Layer Earth, IEEE Trans. on Magnetics, Vol. 28, No. 2, pp. 1497-1500, March 1992. R51-5

[4] P. D. Rancic, L. V. Stefanovic, Dj. R. Djordjevic: An Improved Linear Grounding System Analysis in Two-Layer Earth, IEEE Trans. on Magnetics, Vol. 32, No. 5, pp. 5179-5187, September 1995. R51-5

[5] P. D. Rancic, Z. P. Stajic, Dj. R. Djordjevic, B. S. Tosic: Analysis of Linear Ground Electrodes Placed in Vertical Three - Layer Earth, IEEE Trans. on Magnetics, Vol. 32, No. 3, pp. 1505-1508, May 1996. R51-5