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Identification of a Significant Systematic Error in Pressure Sensor Readings Based on an Autoregressive Model

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Abstract: A new method of identifying the validity of pressure sensor readings has been developed. The method uses duplication of measurements which allows an estimation of the magnitude of an error, although it does not make it possible to establish which sensor is responsible for the error. The method helps to evaluate the systematic error magnitude and to test whether the error exists within a permission range accounting for a correlation structure of measurement series. An autoregressive model with a drift coefficient is applied to investigate a time series of the differences in readings. To test the significance of this coefficient, a modified Student's test is used. Unlike the standard Student's test, this new method tests an interval hypothesis. The null hypothesis assumes the systematic error is within the range and the alternative is out of the range. Error probabilities of type I and type II are calculated. An example of 2nd order autoregressive model was considered and the sensitivity of the proposed method was investigated.

Keywords: Pressure monitoring, Systematic error, Validity of statistical data, Time series analysis, Student's significance test, Interval hypothesis.

1 Introduction

The main problem of a dispatcher of a gas transportation company is to monitor and control the technological regime of a pipeline system section. Failure to comply with the normal operation of the gas transmission system (GTS) can lead to serious economic and environmental consequences, as well as to human casualties. The GTS state is monitored according to information received from the instrumentation. The dispatcher can make right decisions only if the incoming information is valid.

In practice, simple methods are used to control the validity of the readings of the pressure sensors on main gas pipelines, such as checking whether the readings

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and their rate of change are outside the permissible range, etc. [1, 2]. These methods allow one to identify an obvious technical malfunction only.

In [3], the sensor validity has been analyzed without taking into account the redundancy of measurements (the readings time series of every sensor has been considered separately), based on previous measurements only.

In [4 – 6], various approaches to the estimation of the sensor systematic errors have been considered, taking into account their relationship and, as consequence, the redundancy of measurements. But, in these papers, the acceptable threshold of systematic error and the statistical nature of the obtained estimations have not been taken into account.

In [5], the sensor registration problem has been investigated. To solve it, the Kalman filter and EM-algorithm have been applied, and the redundancy of the measurements has been taken into account. As in the other papers, the permissible threshold of systematic error has been not studied.

In [7], a method of the validity control based on groups of “similar” sensors has been proposed. Any influences of the readings of one sensor in a group on other sensors within the group have been investigated. This method does not take in account shot-off valve locations, and the acceptable threshold of systematic error also is not considered.

The systematic error identification problem has been considered in numerous publications dedicated to tracking moving targets [8 – 10]. The duplication of measurements is utilized to estimate biases, but the problem of falling within the acceptable range has not been investigated.

In this paper, it is assumed a small systematic error is not a sign of a sensor malfunction. Therefore, it is necessary to set a certain threshold, the excess of which already indicates a sensor failure. Moreover, the stochastic nature of sensor readings should be considered. The problem of identifying the systematic error in this formulation has not been previously solved. Thus, the novelty of the proposed method, and the validity in its identification, is that a sensor failure is estimated by taking into account the permissible threshold of its systematic error.

In this paper, the problem of identifying sensors with a significant systematic error is considered, and cases where there is no confidence that the error is large remain outside the scope of this study.

The method, proposed in [11], is based on the identification of a significant difference in the readings of hydraulically dependent pressure measuring channels. The hydraulic dependence of readings of the measuring channels is determined not only by the instrumentation location, but also by the current state of the shut-off and control valves. Each group of measuring channels consists of instruments, the pressure between which at a certain point in time cannot differ significantly.

In [12], a method the testing of an interval hypothesis of no difference between mean values of two samples is described. This method is widely used in biostatistics and medicine to comparing of an effectiveness of drugs. The method is described below in this paper in more detail.

However, when comparing the readings of the measuring channels from this group, it is necessary to account for both the inherent error of the instrumentation and the error of signal conversion in the software and hardware of each measuring channel (Fig. 1).

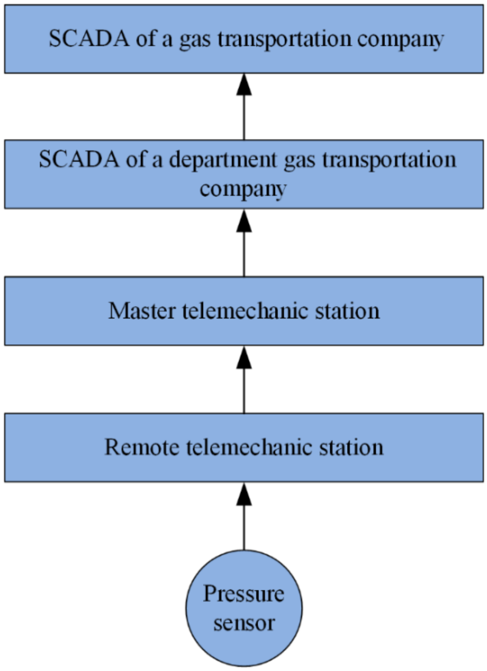


Fig. 1 – *Measuring channel structure.*

The errors of analog circuits and communication lines, as well as the uncertainty of time synchronization, are added to the initial error of the instrumentation. These errors are of a random nature. In this paper, a new statistical method for comparing the readings of the instrumentation measuring channels is proposed.

In the operation of linear telemechanic systems, both data transfer on request with a constant sampling interval and sporadic transfer are used [13]; the measured parameter is transmitted only in case of a significant change in reducing traffic. In the latter case, the sampling interval is variable, and it is necessary to use special methods for the signal analysis. In this paper, the case of a constant sampling interval is studied.

The rest of this paper is organized as follows. The second section describes the *two one-sided t-tests* (TOST) method. In the third section, a probabilistic model of sensor readings and their differences, assuming results of preliminary statistical processing of empirical data, is introduced. The fourth section proposes a new autoregressive model for the identification of a sensor systematic error, assuming a correlation structure of time series. The validity test of sensors is based on a modified Student's test for an interval hypothesis of the model drift coefficient significance. A sensitivity of the proposed new method and its error probabilities of type I and type II are investigated in the fifth section. The sixth section concludes the research.

2 Probabilistic Model of Data Transfer on Request

A probabilistic model for readings of the measuring channel is introduced:

$$P_t = P_t^{true} + \varepsilon_t, \tag{1}$$

where t is the measuring time point, P_t^{true} is the true pressure value in the sensor location and the quantities P_t and ε_t are the signal and the error of the measuring channel, respectively.

The case when there are only two measuring channels in the group, $P_t^{(a)}$ and $P_t^{(b)}$, is considered. The difference between the readings of the channels is determined by their errors only:

$$e_t = P_t^{(a)} - P_t^{(b)} = P_t^{true} - P_t^{true} + \varepsilon_t^{(a)} - \varepsilon_t^{(b)} = \varepsilon_t^{(a)} - \varepsilon_t^{(b)}. \tag{2}$$

Figs. 2 and 3 show examples of time series e_t, calculated by real data.

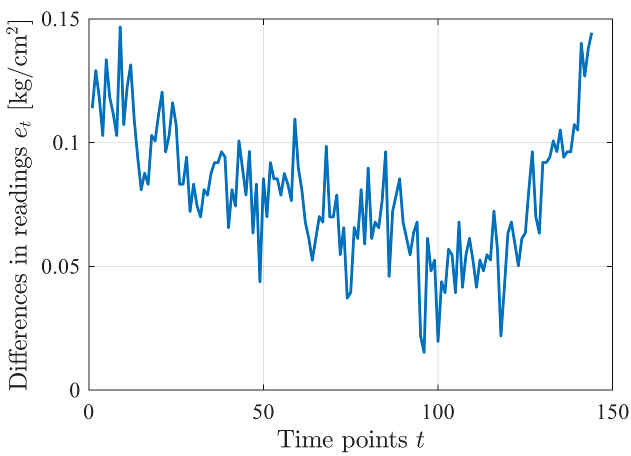


Fig. 2 – *An example of nonstationary time series* e_t.

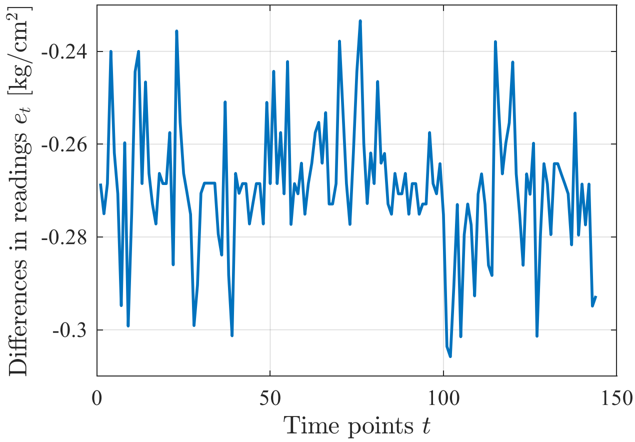


Fig. 3 – *An example of stationary time series* e_t.

The measuring data of 29 sensor groups of the linear part of the main gas pipeline LLC "Gazprom transgaz Moscow" has been investigated. For all the data, the hypothesis of stationarity was analyzed with the autocorrelation function (ACF) and the Dickey-Fuller test [14]. In most cases, specifically 24 of 29 (83%), the differences e_t were stationary time series. For other cases, the very fact of nonstationarity, apparently, indicates the invalidity of the readings; however, this requires a separate study. In this paper, only stationary time series were considered.

3 Two One-sided T-tests (TOST) Procedure

In 1987, Schuirmann proposed a method to test interval hypotheses [12], i.e., whether a difference of means is into an interval of $[\theta_1, \theta_2]$. Let's present the method by changing some of the notation. A test sample and a reference one of independent normal observations are considered. The null hypothesis is formulated as

$$H_0:\ \Delta\mu \notin [\theta_1, \theta_2] \tag{3}$$

and the alternative is

$$H_1:\ \Delta\mu \in [\theta_1, \theta_2], \tag{4}$$

where $\Delta\mu = \mu_R - \mu_T$ is a difference of the unknown means of the test sample and the reference one.

The TOST procedure provides the tests of two one-sided hypotheses:

$$H_0^-:\ \mu \le \theta_1, \quad H_1^-:\ \Delta\mu > \theta_1 \tag{5}$$

and

$$H_0^+:\ \Delta\mu \ge \theta_2, \quad H_1^+:\ \Delta\mu < \theta_2. \tag{6}$$

The means are considered as equivalent if

$$t_1 = \frac{\left(\bar{x}_R - \bar{x}_T\right) - \theta_1}{\hat{\sigma}} \ge t_{1-\alpha} \tag{7}$$

and

$$t_2 = \frac{\theta_2 - \left(\bar{x}_R - \bar{x}_T\right)}{\hat{\sigma}} \ge t_{1-\alpha}, \tag{8}$$

where $\bar{x}_R$ and $\bar{x}_T$ are the sample estimations of the unknown means, $\hat{\sigma}$ is the sample estimation of the standard deviation of $\bar{x}_R - \bar{x}_T$, which is calculating differently depending on whether variances of samples are assumed to be different or the same, $t_{1-\alpha}$ is the quantile of the Student's t-distribution with ν degree of freedom and α is the nominal level of significance. The real probability of the type I error was not evaluated in [12], but it's stated that the TOST is "identical to the procedure of declaring equivalence only if the ordinary $1-2\alpha$ confidence interval for $\Delta\mu$ is completely contained in the equivalence interval $\left[\theta_1, \theta_2\right]$".

There is some incorrectness in the procedure description [12]. The null hypothesis cannot be formulated as an inequality, because in this case the test statistic distribution depends on the unknown value of $\Delta\mu$. It is necessary to reformulate the one-sided hypotheses:

$$H_0^-:\ \Delta\mu = \theta_1, \quad H_1^-:\ \Delta\mu > \theta_1, \tag{9}$$

and

$$H_0^+:\ \Delta\mu = \theta_2, \quad H_1^+:\ \Delta\mu < \theta_2. \tag{10}$$

In this case, both statistics t_1 and t_2 would have a Student's t-distribution if the corresponding null hypothesis, H_0^- or H_0^+, is true.

One can consider an alternative TOST-procedure, by assuming the null hypothesis as $H_0:\ \Delta\mu \in \left[\theta_1,\ \theta_2\right]$ and the alternative as $H_1:\ \Delta\mu \notin \left[\theta_1,\ \theta_2\right]$. Let's compare this variation with the original one. Consider the statistic:

$$t = \frac{\bar{x}_R - \bar{x}_T}{\hat{\sigma}}. \tag{11}$$

It has a non-central Student's distribution with ν degree of freedom and the non-centrality parameter $\Delta\mu/\sigma$, where σ is the real (but unknown) standard deviation of $\bar{x}_R - \bar{x}_T$. If the sample sizes are large enough (50 observations or more), then t is approximately normal with mean $\Delta\mu/\sigma$ and standard deviation 1. Because σ is unknown, it should be replaced by its estimation $\hat{\sigma}$. If H_0^- is true, then $\mathbf{E}t = \tilde{\theta}_1 = \theta_1/\hat{\sigma}$, if H_0^+ is true, then $\mathbf{E}t = \tilde{\theta}_2 = \theta_2/\hat{\sigma}$. Fig. 4 allows to compare the variants of TOST null hypotheses.

Fig. 4 shows areas for equivalent and non-equivalent means for two variants of the null hypothesis: $H_0: \Delta\mu \in [\theta_1,\ \theta_2]$ and $H_0: \Delta\mu \notin [\theta_1,\ \theta_2]$.

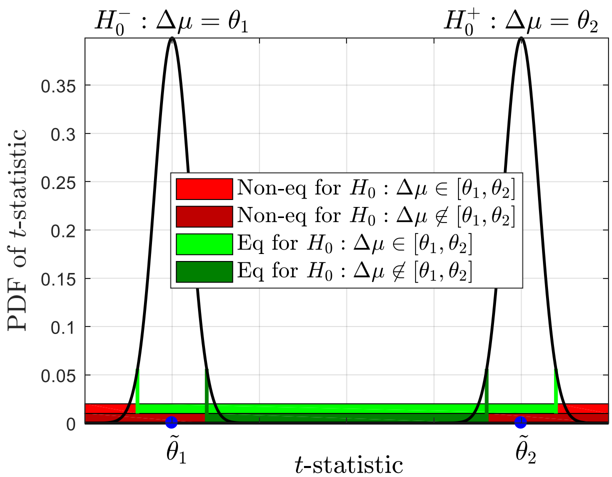


Fig. 4 – *Comparing two variants of TOST null hypotheses.*

In this paper, the equivalence test to detect a significant systematic error, which is a signal of sensor readings invalidity is applied. The original TOST would detect the invalidity more strictly, by discarding the samples, in which the equivalence was not confirmed. The alternative TOST is less strict in this sense, because it discards only samples, in which the non-equivalence is confirmed. This thin difference is obviously illustrated by Fig. 4.

If $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are close to zero (it can be due to small sample sizes, large variances or small θ_1 and θ_2), using the original TOST-procedure with the null hypothesis $H_0: \Delta\mu \notin [\theta_1, \theta_2]$, the means will be considered non-equivalent even if the sample value of t-statistic is exactly zero. The alternative procedure with the null hypothesis $H_0: \Delta\mu \in [\theta_1, \theta_2]$ leads to the more wider equivalence area.

It is not enough to only use the TOST-procedure to detect a significant systematic error of pressure sensors. The correlation structure of the pressure sensors time series has to be considered.

4 Systematic Error Identification

Let us consider the most difficult to detect type of invalidity – the systematic error of the measuring channel, the magnitude of which is large enough to mislead the dispatcher. However, it is not outside the permissible limits that would cause the readings to be rejected by known algorithms (Fig. 5). Therefore, this type of invalidity cannot be determined by existing monitoring methods [1].

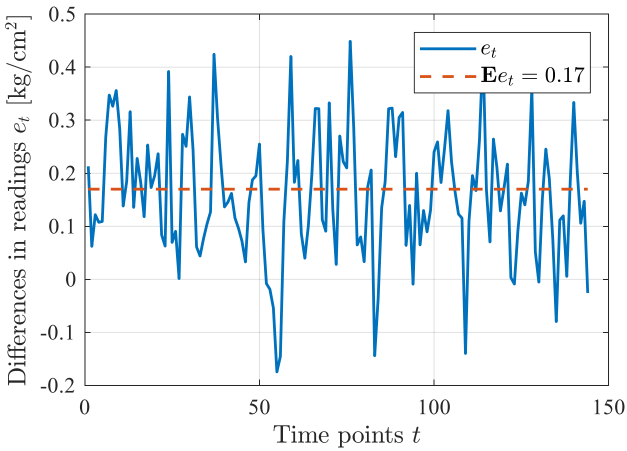


Fig. 5 – *An example of measurements with systematic error* $\mathbf{E}e_t = 0.17$.

Ideally, a measuring device should not have a systematic error, but in fact, a small systematic measurement error is acceptable. For practical purposes, it is necessary to estimate the error magnitude and determine whether it is permissible.

However, it is impossible to determine the error magnitude of each measuring channel based on the difference of the readings only. There are cases when large errors of both channels have the same signs and compensate each other. Nevertheless, by analyzing the readings differences, it is possible to identify a wide class of invalidities, for example, a large systematic error of one of the channels, if the other is serviceable.

In terms of mathematical statistics, the systematic error is an expectation (mean) of the measuring error. To calculate it, write down the expectation of the readings difference:

$$\mathbf{E}e_t = \mathbf{E}\varepsilon_t^{(1)} - \mathbf{E}\varepsilon_t^{(2)}. \tag{12}$$

Based on a dispatcher's expert opinion, it is possible to estimate the threshold value of $\mathbf{E}e_t$, which is denoted by Δ, and check the hypothesis of the following:

$$H_0 : |\mathbf{E}e_t| \le \Delta, \quad H_1 : |\mathbf{E}e_t| > \Delta \,. \tag{13}$$

Here H_0 is a null hypothesis and H_1 is an alternative one.

If the alternative hypothesis is not rejected, then it means unambiguously that there is an intolerably large systematic error in one of the channels or in them both simultaneously. If the null hypothesis is not rejected, it still doesn't mean the readings are valid, but it can be assumed that it is very likely.

The observations e_t are dependent; therefore, the standard Student's test of the means' equality cannot be applied. Nonparametric tests of location such as Wilcoxon, Van der Van or median tests are limited to testing the class of simple hypothesis $H_0 : \mathbf{E}e_t = 0, \quad H_1 : \mathbf{E}e_t \ne 0$, provided that the measurements are independent [15].

The null hypothesis H_0 with the two-sided alternative can be replaced by two hypotheses with one-sided alternatives:

$$H_0^+ : \mathbf{E}e_t = \Delta, \quad H_1^+ : \mathbf{E}e_t > \Delta \,, \tag{14}$$

$$H_0^- : \mathbf{E}e_t = -\Delta, \quad H_1^- : \mathbf{E}e_t < -\Delta \,. \tag{15}$$

If both null hypotheses, H_0^+ and H_0^-, are not rejected, the general null hypothesis H_0, (13), should not be rejected either. If one of the null hypotheses, H_0^+ or H_0^-, are rejected, the general null hypothesis H_0 (the systematic error is zero) should be rejected.

To test the statistical hypotheses (14) and (15) it is necessary to account the correlation structure of a time series e_t; that's why the Box-Jenkins method for its modeling is used [16].

A statistical study of the stationary available data shows, that the ACF decreases exponentially and the partial ACF values are statistically insignificant after a certain lag. Therefore, the considering time series correspond to the autoregressive models.

To account for a possible drift of the differences in reading an autoregressive model, a constant a_0 is applied:

$$e_t = a_0 + a_1 e_{t-1} + a_2 e_{t-2} + \cdots + a_k e_{t-k} + u_t \,, \tag{16}$$

where $a_0, a_1, \ldots, a_k$ are model coefficients, u_t is normal white noise with variance $\mathbf{D}u_t = \sigma^2$, and k is an autoregressive order. A statistical analysis of 24 actual time series shows, that k can be from 0 to 3. The value of k was determined by model residuals testing corresponding to the white noise properties using the Box-Jenkins method [16].

Let us reformulate hypotheses (14) and (15) taking into account the fact that the time series of the differences in readings corresponds to the autoregressive model. The expectation of the differences in readings is

$$\mathbf{E}\,e_t = \frac{a_0}{1-\sum_{i=1}^{k} a_i}. \tag{17}$$

Therefore, (14) and (15) take the form

$$H_0^+ :\ \frac{a_0}{1-\sum_{i=1}^{k} a_i} = \Delta, \quad H_1^+ :\ \frac{a_0}{1-\sum_{i=1}^{k} a_i} > \Delta, \tag{18}$$

$$H_0^- :\ \frac{a_0}{1-\sum_{i=1}^{k} a_i} = -\Delta, \quad H_1^- :\ \frac{a_0}{1-\sum_{i=1}^{k} a_i} < -\Delta. \tag{19}$$

To simplify testing hypotheses (18) and (19), let us introduce two additional variables, $e_t^{(1)} = e_t - \Delta$ and $e_t^{(2)} = e_t + \Delta$, shifted relative to the original series by the threshold value Δ in positive and negative directions. Then, the hypotheses take the form:

$$H_0^+ :\ \mathbf{E}e_t^{(1)} = 0, \quad H_1^+ :\ \mathbf{E}e_t^{(1)} > 0, \tag{20}$$

$$H_0^- :\ \mathbf{E}e_t^{(2)} = 0, \quad H_1^- :\ \mathbf{E}e_t^{(2)} < 0, \tag{21}$$

$$H_0^+ :\ \frac{a_0^{(1)}}{1-\sum_{i=1}^{k} a_i} = 0, \quad H_1^+ :\ \frac{a_0^{(1)}}{1-\sum_{i=1}^{k} a_i} > 0, \tag{22}$$

$$H_0^- :\ \frac{a_0^{(2)}}{1-\sum_{i=1}^{k} a_i} = 0, \quad H_1^- :\ \frac{a_0^{(2)}}{1-\sum_{i=1}^{k} a_i} < 0, \tag{23}$$

where $a_0^{(i)}$ corresponds to $e_t^{(i)},\ i \in \{1,2\}$.

Since the stationary time series are to be considered, one has that $\sum_{i=1}^{k} a_i < 1$; therefore, the denominators of the fractions (20) – (23) are positive. Therefore, the final form of the considered hypotheses is:

$$H_0^+ :\ a_0^{(1)} = 0, \quad H_1^+ :\ a_0^{(1)} > 0, \tag{24}$$

$$H_0^- :\ a_0^{(2)} = 0, \quad H_1^- :\ a_0^{(2)} < 0. \tag{25}$$

The autoregressive coefficients $a_0^{(1)}$ and $a_0^{(2)}$ should be estimated by actual data. For estimating, the least square method is recommended [17], because in this case the distribution of t-statistic is more stable than when using the maximum likelihood method. In addition, the least square method is significantly faster, although it is less efficient.

When using the least square method, the autoregressive coefficients' estimations and their covariance matrix are

$$\boldsymbol{a} = \left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}\boldsymbol{X}^T\boldsymbol{e}, \tag{26}$$

$$\boldsymbol{C} = \mathrm{cov}\left(\boldsymbol{a}\right) = \sigma^2\boldsymbol{V}^{-1}, \tag{27}$$

where

$$\boldsymbol{V} = \mathbf{E}\left(\boldsymbol{X}^T\boldsymbol{X}\right) = (n-k)\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \gamma_0 & \gamma_1 & \gamma_2 & \cdots & \gamma_{k-1} \\ 0 & \gamma_1 & \gamma_0 & \gamma_1 & \cdots & \gamma_{k-2} \\ 0 & \gamma_2 & \gamma_1 & \gamma_0 & \cdots & \gamma_{k-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \gamma_{k-1} & \gamma_{k-2} & \gamma_{k-3} & \cdots & \gamma_0 \end{bmatrix}, \tag{28}$$

$\boldsymbol{X}$ is a least square model matrix, $\boldsymbol{e}$ is a vector of last $n-k$ values of the series e_t, γ_i are autocovariances, $\gamma_i = \mathrm{cov}\left(e_t, e_{t-i}\right)$, and σ^2 is a noise variance.

Estimating the coefficients, σ^2 and γ_i are unknown; instead, two variants of covariance matrix estimation can be used to calculate t-statistics: $\hat{\boldsymbol{C}} = \hat{\sigma}^2\left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}$ or $\hat{\boldsymbol{C}} = \hat{\sigma}^2\hat{\boldsymbol{V}}^{-1}$. The corner element of the matrix $\hat{\boldsymbol{C}}$ is a variance estimate of the model constant. In the second case, it is calculated as

$$\hat{c}_{0,0} = \frac{\hat{\sigma}^2}{n-k}, \quad \hat{\sigma}^2 = \frac{\boldsymbol{r}^T\boldsymbol{r}}{n-k}, \tag{29}$$

where $\boldsymbol{r}$ are residuals of the model.

To obtain the matrix $\hat{\boldsymbol{V}}$, it is necessary to calculate empirical autocovariances $\hat{\gamma}_i$.

The t-statistics for hypotheses (24) – (25) can be obtained as

$$t_1 = \frac{\hat{a}_0^{(1)}}{\sqrt{\hat{c}_{0,0}}} = \frac{\hat{a}_0^{(1)}}{\hat{\sigma}}\sqrt{n-k}, \quad t_2 = \frac{\hat{a}_0^{(2)}}{\sqrt{\hat{c}_{0,0}}} = \frac{\hat{a}_0^{(2)}}{\hat{\sigma}}\sqrt{n-k}. \tag{30}$$

One has that the least square method using t-statistics (30) have a non-central Student's distribution with $n-2k-1$ degrees of freedom (DOF) [18], which for large samples is well approximated by the normal law. The non-centrality parameters are equal to the expectations of the t-statistics, which in this case are

$$\mathbf{E}t_1 = \left(\mathbf{E}e_t - \Delta\right)\left(1 - \sum_{i=1}^{k} a_i\right)\frac{\sqrt{n-k}}{\sigma}, \tag{31}$$

$$\mathbf{E}t_2 = \left(\mathbf{E}e_t + \Delta\right)\left(1 - \sum_{i=1}^{k} a_i\right)\frac{\sqrt{n-k}}{\sigma}. \tag{32}$$

To test the hypotheses (24) – (25), it is necessary to fix a significance level ϕ and calculate the quantile of the classical (central) Student's distribution $t_{1-\phi}$.

If $t_1 > t_{1-\phi}$, the null hypothesis H_0^+ is rejected.

If $t_2 < -t_{1-\phi}$, the null hypothesis H_0^- should be rejected.

In the case of rejecting one of the null hypotheses, H_0^+ or H_0^-, the general null hypothesis H_0 also should be rejected.

Moreover, if $\hat{a}_0^{(1)} > 0$ (or $\hat{a}_0^{(2)} < 0$), then there is no need to test the hypothesis H_0^- (or H_0^+).

5 Simulation Example

To illustrate the operation of this method, let us consider an example of the autoregressive model of 2nd order for the differences in readings (Fig. 5).

The model parameters are given $a_0 = 0.12,\ a_1 = 0.5,\ a_2 = -0.2$, the noise variance is $\sigma^2 = 0.01$, and a sample of size $n = 144$ was simulated. Taking the threshold value to be $\Delta = 0.1\,\text{kg}/\text{cm}^2$ and a significance level of $\phi = 0.05$, the results shown in **Table 1** are obtained.

Table 1

Calculation results.

Parameter	Value
Hypothesis	H_0^+
$\hat{a}_0^{(1)}$	0.042
$\hat{\sigma}$	0.1
n	144
k	3

Parameter	Value
Statistic t_1 DOF	$n - 2k - 1 = 137$
Statistic t_1	5.06
$T_{0.95}(137)$	1.66
Test result	rejected, $t_1 > t_{0.95}(137)$

6 Sensitivity of the Method

Let us investigate the sensitivity of the developed method using this example and study how α and β, the error probabilities of type I and type II, depend on the true systematic error.

Note, ϕ is not equal to a type I error probability for the interval hypothesis. This probability is not a constant in this case.

Regardless of which of the hypotheses H_0 or H_1 is true, the statistics t_1 и t_2 have a non-central Student's distribution with $n-2k-1$ degrees of freedom and non-centrality parameter ν, which is equal to $\nu \approx \mathbf{E}t_1$ for t_1 and $\nu \approx \mathbf{E}t_2$ for t_2. The expectations are calculated by the formulas (31) and (32), respectively. For large samples, non-central Student's distribution can be approximate with the normal law, $G(t,\nu) \approx \Phi(t_{1-\varphi} - \nu)$, where $G(t,\nu)$ is the non-central Student's cumulative distribution function and $\Phi(x)$ is the same for standard normal distribution.

A type I error arises in case, if the null hypothesis H_0 is true, i.e. $e_t \in [-\Delta; \Delta]$, and at least one of the statistics t_1 or t_2 hits critical region, i.e. $t_1 > t_{1-\phi}$ or $t_2 < -t_{1-\phi}$. Let's find the probabilities of these events:

$$\alpha_1 = \mathbf{P}_{H_0}\left\{t_1 > t_{1-\phi}\right\} = 1 - G\left(t_{1-\phi}, \mathbf{E}t_1\right) \approx 1 - \Phi\left(t_{1-\phi} - \mathbf{E}t_1\right), \tag{33}$$

$$\alpha_2 = \mathbf{P}_{H_0}\left\{t_2 < -t_{1-\phi}\right\} = G\left(-t_{1-\phi}, \mathrm{E}t_2\right) \approx \Phi\left(-t_{1-\phi} - \mathbf{E}t_2\right). \tag{34}$$

The events $\{t_1 > t_{1-\phi}\}$ and $\{t_2 < -t_{1-\phi}\}$ are incompatible, therefore the general type I error probability is

$$\alpha = \mathbf{P}_{H_0}\left\{t_1 > t_{1-\phi} \text{ or } t_2 < -t_{1-\phi}\right\} = \alpha_1 + \alpha_2. \tag{35}$$

It should be noted that it is impossible to reject both null hypothesis H_0^+ and H_0^- versus considered alternatives, H_1^+ and H_1^-, therefore there is no need to consider multiple testing problem [19].

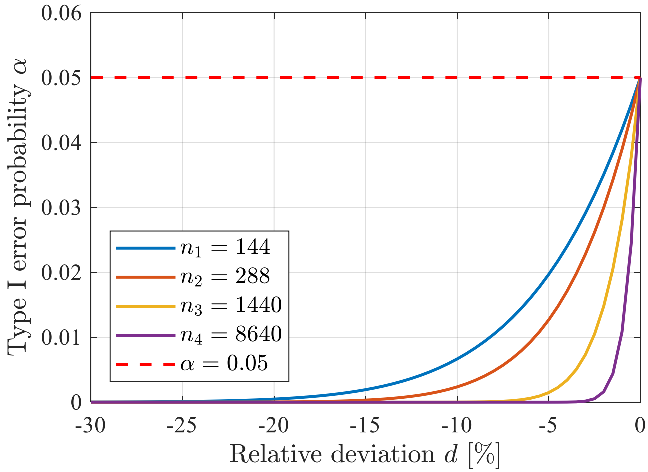
A type II error arises if the null hypothesis is false, i.e., $\mathrm{E}\,e_t \in (-\infty; -\Delta) \cup (\Delta; \infty)$, but the TOST procedure doesn't reject it, i.e., $t_1 < t_{1-\phi}$ and $t_2 > -t_{1-\phi}$ simultaneously. The probability of this error is:

$$\begin{aligned}\beta &= \mathbf{P}_{H_1}\left\{t_1 < t_{1-\phi} \text{ and } t_2 > -t_{1-\phi}\right\} = 1 - \mathbf{P}_{H_1}\left\{t_1 > t_{1-\phi} \text{ or } t_2 < -t_{1-\phi}\right\} = \\ &= G\left(t_{1-\phi}, \mathbf{E}t_1\right) - G\left(-t_{1-\phi}, \mathbf{E}t_2\right) \approx \Phi\left(t_{1-\phi} - \mathbf{E}t_1\right) - \Phi\left(-t_{1-\phi} - \mathbf{E}t_2\right).\end{aligned} \tag{36}$$

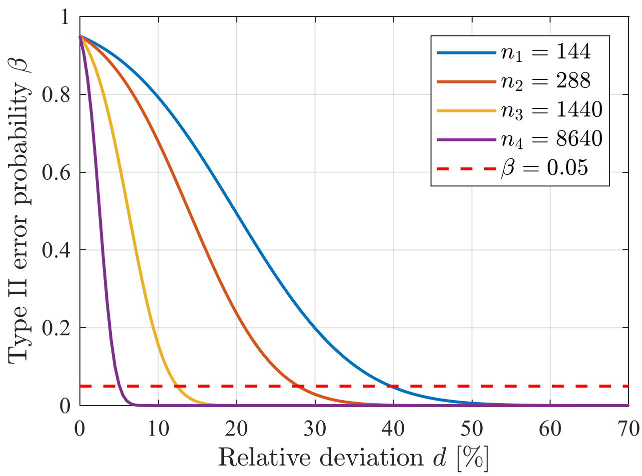
One might get the impression that $\alpha+\beta=1$, but the probabilities α and β are calculated with different assumptions: α assuming $\mathbf{E}e_t \in [-\Delta; \Delta]$ and β assuming $\mathbf{E}e_t \in (-\infty; -\Delta) \cup (\Delta; \infty)$. Therefore, the expression has no sense.

As the problem is symmetric about zero, it is natural to consider a probability error dependency with respect to the relative deviation of the absolute value of the systematic error from the threshold value:

$$d = \left(\left|\mathbf{E}e_t\right| - \Delta\right) / \Delta. \tag{37}$$



(a)



(b)

Fig. 6 – *Dependency of the error probabilities of type I (a) and type II (b) on the deviation of the systematic error from its threshold.*

Figs. 6a and 6b show the probabilities α and β calculated for various sample sizes: $n_1 = 144$, $n_2 = 288$, $n_3 = 1440$, $n_1 = 8640$.

Let us fix the type II error probability $\beta = 0.05$ and find values of the deviation *d* of the systematic error from its threshold, which can be identified by this method (**Table 2**). Notice, the type I error probability is always less than the significance level, $\alpha \le \phi = 0.05$; therefore, there is no need to take this into account when choosing the sample size.

As one can see from **Table 2**, to identify a 5% deviation of the systematic error from the threshold with a probability of 5%, it is necessary to have more than 8640 observations.

Table 2
Deviation of the systematic error from its threshold.

n	$\lvert \mathbf{E} e_t \rvert - \Delta$ [kg/cm^2]	d [%]
144	0.0400	40.0
288	0.0280	28.0
1440	0.0125	12.5
8640	0.0050	5.0

7 Calculation Algorithm

Sometimes there is a possibility to obtain additional measurements from pressure sensors, in this case one should adapt the necessary sample size. For this it is required:

- to obtain from an expert the threshold value of the systematic error Δ;
- to choose the nominal significance level φ and the permission level of the type II error probability β;
- to choose an acceptable deviation (absolute or relative) of the absolute value of the real systematic error from its threshold;
- to estimate a sample size providing an identification of this deviation with given error probabilities;
- to calculate the differences in readings to get a time series sample of the given size.

If it is impossible to get new measurements and there is a sample with fixed size, one can estimate a maximum value of the deviation *d* at fixed probability β.

Then one should:

– establish if the sequence of the differences is a stationary time series with standard methods (ACF plot and Dickey-Fuller test);
– evaluate the autoregressive model order and estimate the model parameters with the least square method;
– verify the model by testing whether the residuals possess white noise characteristics;
– test the hypotheses (24) – (25) with the proposed modification of the Student's test.

If the general null hypothesis H_0 is rejected, there is too large of a systematic error on one of the sensors or both at once and the measured data is invalid. Otherwise, one can assume the sensors are serviceable and the data is accurate enough.

8 Concluding Remarks

The proposed method allows one to identify systematic errors by taking into account a permissible threshold and the data correlation structure, which is to be researched only in the case of stationarity of time series of sensor readings. The real data analysis has shown that in most cases (more precisely, 83%) the time series are stationary.

For the stationary case it is possible to distinguish a small acceptable error from a large one. To achieve this goal, an interval hypothesis was considered and tested by the modified Student's test.

Testing of the interval two-sided hypothesis was reduced to a testing of the two simple one-sided hypotheses of the significance of the drift coefficient of the autoregressive model.

The sensitivity of the method was studied. The probabilities of errors of types I and II were calculated and the required sample size was estimated.

Finally, let us summarize by repeating that the proposed method allows one to identify a large systematic error of sensor readings and to prevent wrong decisions in the operational control of gas transmission.

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